

Development and Testing of Airfoils for High-Altitude Aircraft

Final Report

for the Period
September 1, 1994 – May 31, 1996

100-300

200-300

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June 1996

1 Completed Work

Essentially all the work proposed in the proposal has been completed or adequately addressed. Additional work on heat-exchanger aerodynamics not in the original proposal was also conceived and executed with the agreement of the Technical Monitor.

A summary of the specific tasks accomplished is given below. The corresponding detailed results have been transmitted to Dryden personnel via e-mail as soon as they became available. All e-mail transmissions have been saved, and the key ones are given in the Appendices, along with relevant plots, figures, and papers which were generated.

1.1 Airfoil design

The Apex-16 airfoil was designed specifically for the APEX test vehicle. It is intended to be representative of airfoils required for lightweight aircraft operating at extreme altitudes, which is the primary research objective for the APEX program. In the course of designing this airfoil, Fairly extensive studies were made over the Mach number and Reynolds number ranges of interest. Also considered were the airfoil thickness, pitching moment, and off-design behavior. Limited use was made of the optimization driver LINDOP [1] to fine-tune the final design and resolve the numerous conflicting requirements. Appendix B shows the Apex-16 airfoil geometry, coordinates, and computed performance polars.

1.2 Study of airfoil constraints on pullout maneuver

The maximum ceiling parameter $M^2 C_L$ value achievable by the Apex-16 airfoil (or any airfoil for that matter) was found to be a strong constraint on the pullout maneuver. It was concluded that if data is to be acquired in level flight, then some sort of lift augmentation would be required, since any airfoil has little or no excess-lift capability at its ceiling condition (by definition). A workable alternative approach which was identified is to use wing lift to achieve pullout, but then acquire data in windup turns to achieve the target C_L .

1.3 Selection of tail airfoils

Several candidate airfoils for the tail were examined. The primary goal was predictable behavior at low Reynolds numbers, and good tolerance to flap deflections. The NACA 1410 and 2410 airfoils (inverted) were identified as good candidates.

1.4 Examination of wing twist

A number of issues related to wing twist were examined. These included: simplicity, obtaining a uniform local c_t across the test section, obtaining a high overall $C_{L\max}$

for the pullout, avoiding tip stall. Both washin and washout options were examined computationally. In the end it was decided that a simple flat wing was a reasonable compromise between all the requirements, and allowed the use of one mold for both wing halves — a significant cost saving.

1.5 Test section instrumentation and layout

It was decided that the instrumentation for the test section would consist of surface pressure taps, wake rakes, surface-mounted microphones, and skin-friction gauges. Using multiple wake rakes was deemed desirable to verify spanwise uniformity, but was found to be too demanding of the limited data bandwidth available. Using a single wake rake mounted on a mechanical traverse was rejected due to weight and complexity. It was decided to use a single fixed wake rake for accurate measurement of the wake momentum defect, and multiple integrating rakes to verify spanwise uniformity. Several test section layouts were designed, with varying degrees of compactness. One of these was selected by Dryden personnel for use on APEX.

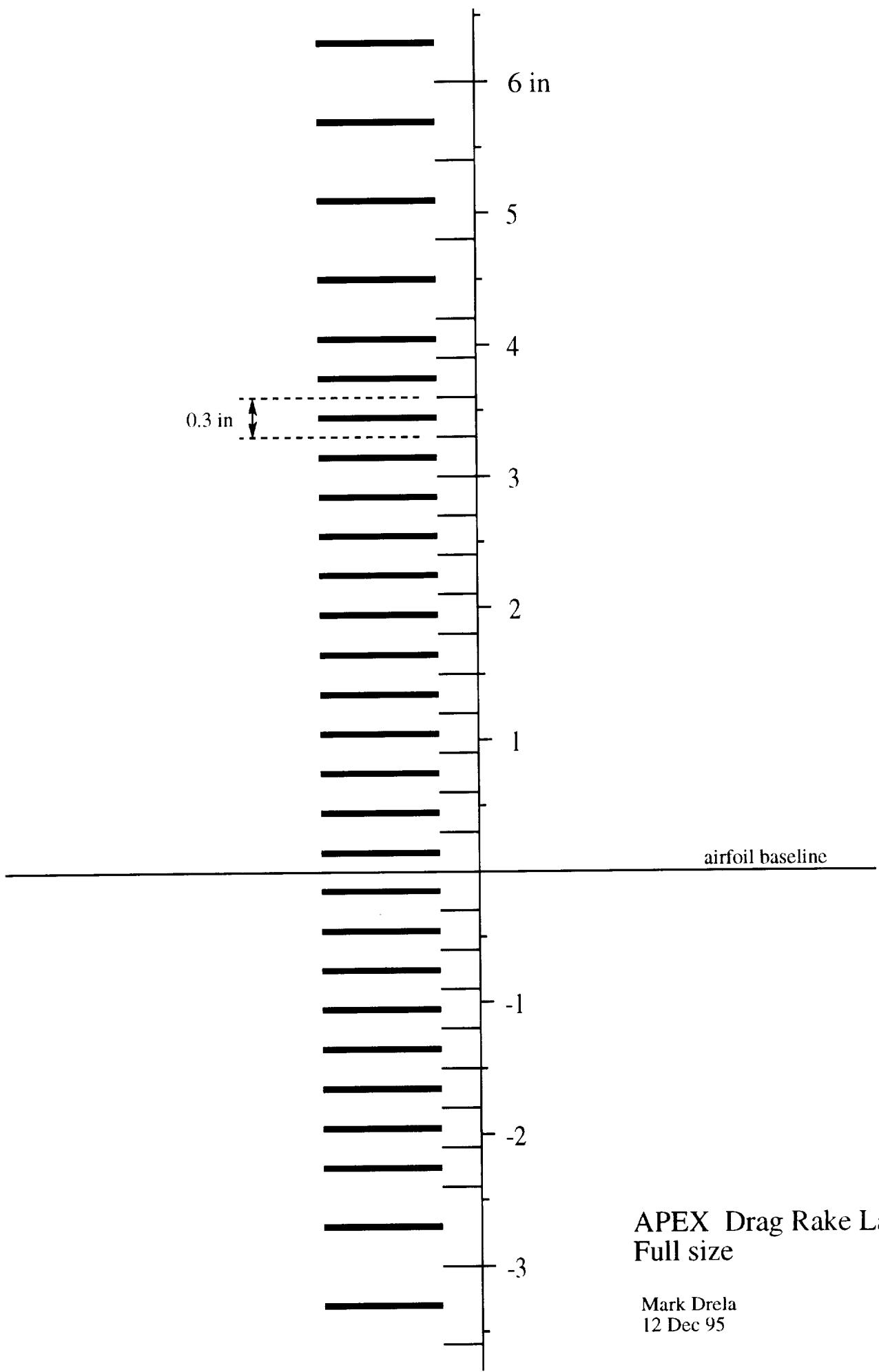
The wake rake was designed specifically to capture the wakes anticipated at test altitudes. A design guide for the integrating rakes was also prepared. The layouts and design guide are attached as Appendices B and C.

1.6 Integrated airfoil/heat-exchanger tests

A modest wind tunnel test was performed for an integrated airfoil/heat-exchanger configuration, which is currently on Aurora's *Theseus* aircraft. Although not directly related to the APEX tests, the aerodynamics of heat exchangers has been identified as a crucial aspect of designing high-altitude aircraft, and hence is relevant to the ERAST program. The computational studies and tunnel tests have proven the validity of integrating the heat exchanger with the wing airfoil to achieve surprisingly low drag levels at low Reynolds numbers. The results have already appeared in the Jan-Feb '96 issue of the *Journal of Aircraft* [2]. This paper is attached as Appendix D.

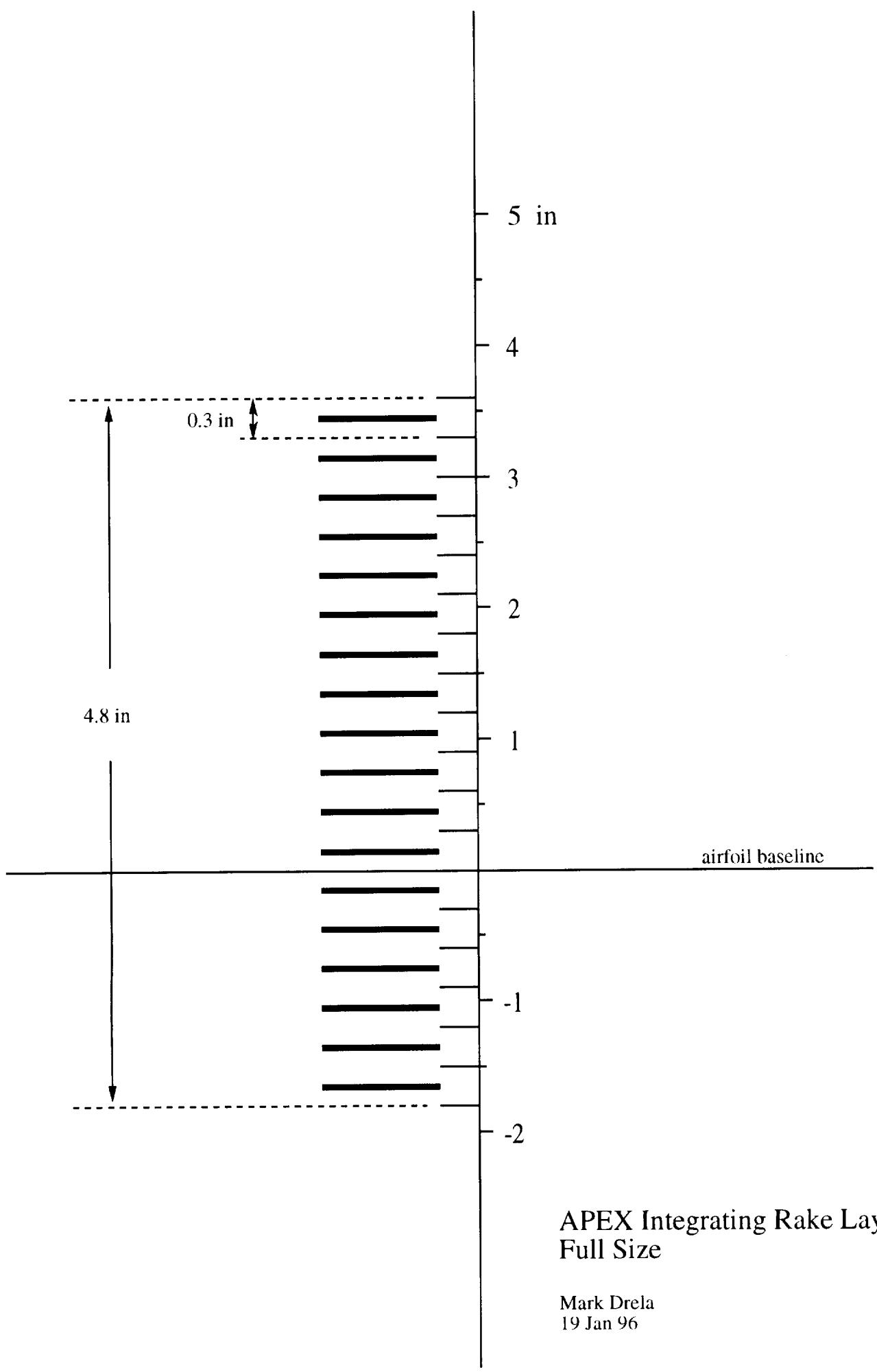
References

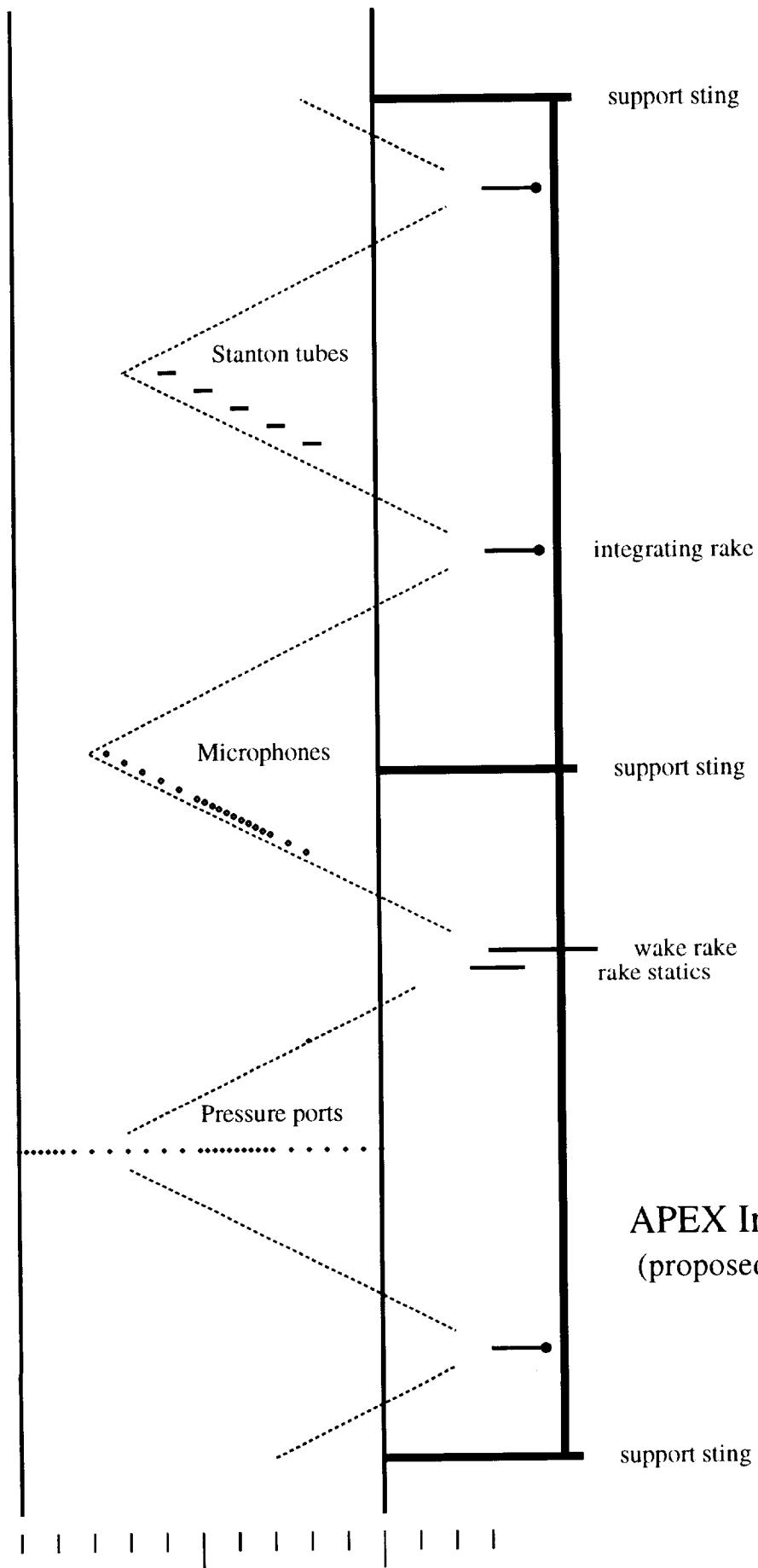
- [1] M. Drela. Design and optimization method for multi-element airfoils. AIAA Paper 93-0969, Feb 1993.
- [2] M. Drela. Aerodynamics of heat exchangers for high-altitude aircraft. *Journal of Aircraft*, 33(2), Mar-Apr 1996.



APEX Drag Rake Layout
Full size

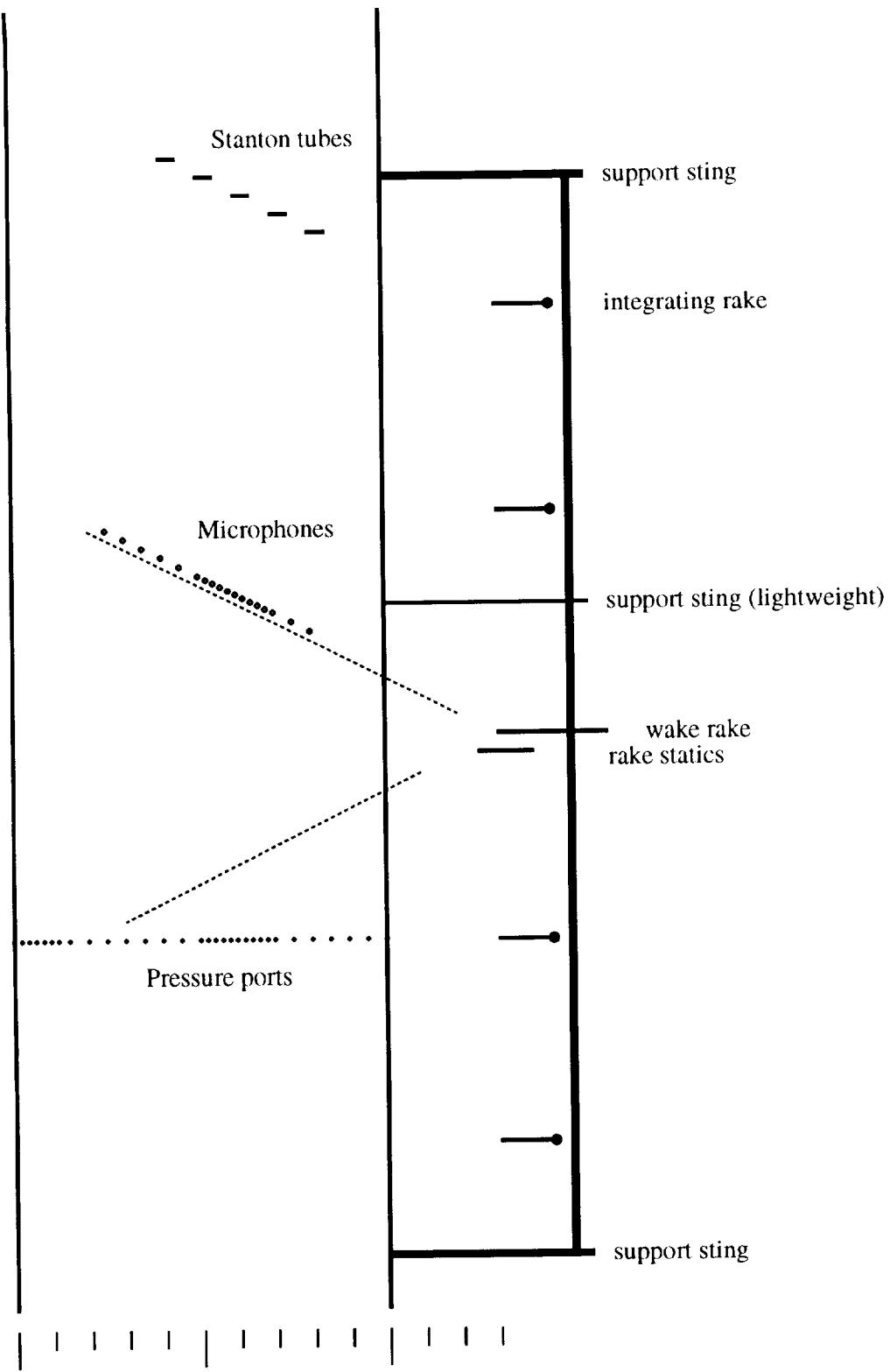
Mark Drela
12 Dec 95





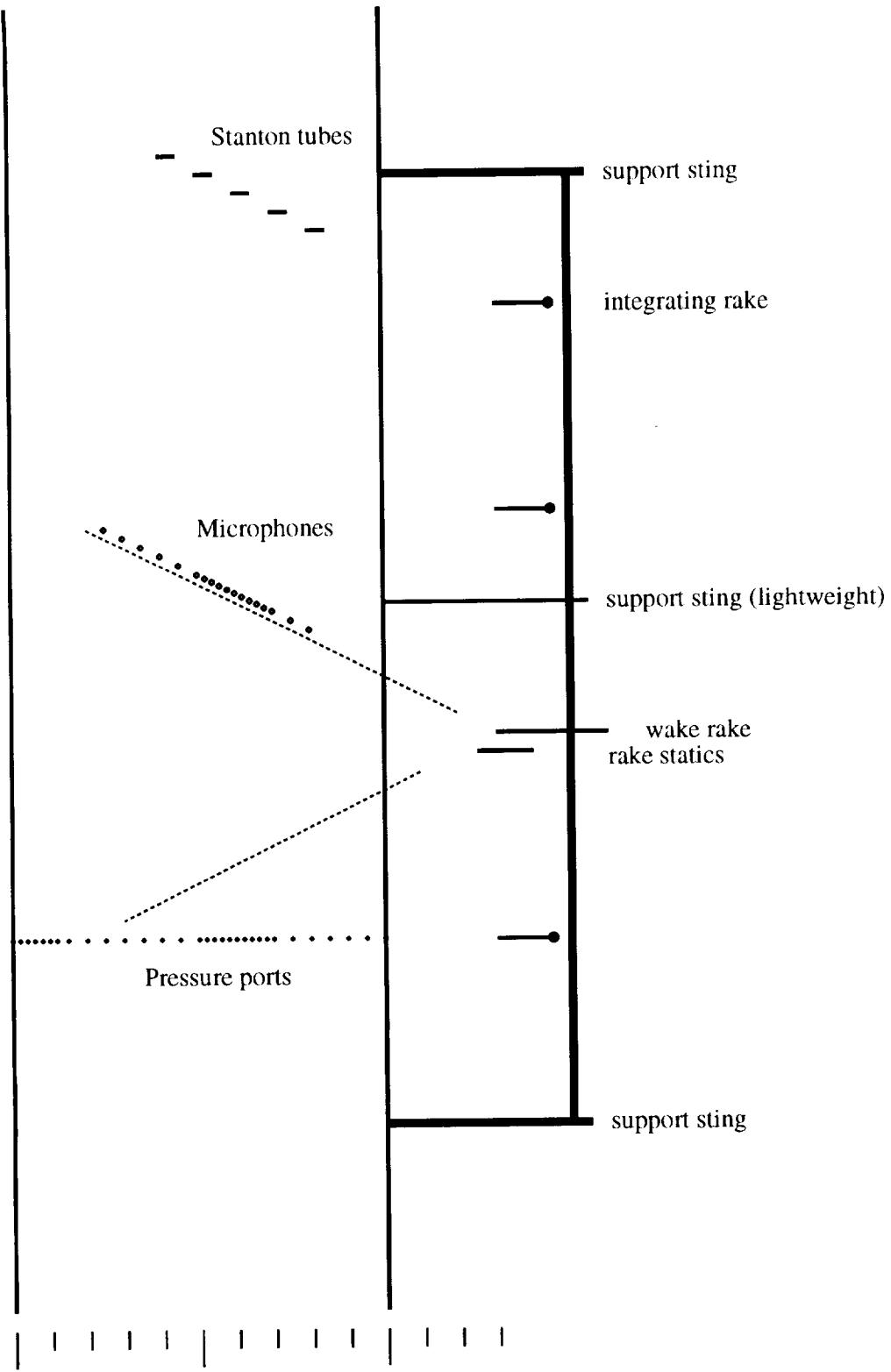
APEX Instrumentation Layout
(proposed)

Mark Drela
7 Feb 96



APEX Instrumentation Layout II
(proposed)

Mark Drela
9 Feb 96



APEX Instrumentation Layout III
(proposed)

Mark Drela
9 Feb 96

Appendix C: Integrating Rake Design Document

Integrating Rake Design

Mark Drela, MIT Aero & Astro
10 Dec 95

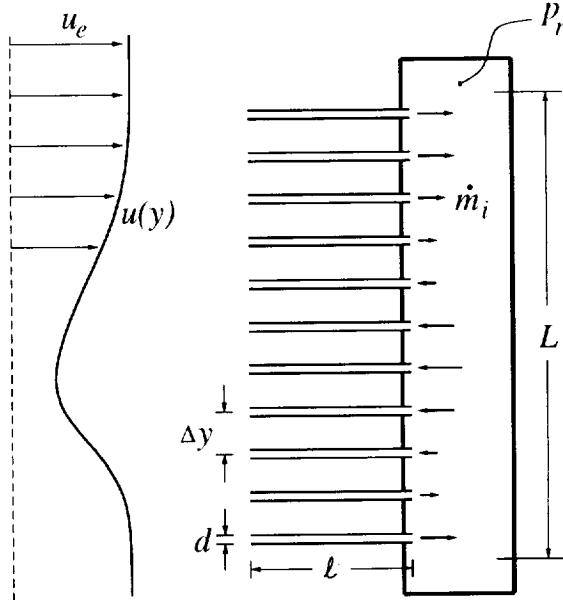


Figure 1: Integrating rake layout and dimensions

1 Basic Relations

An integrating rake consists of an array of pitot tubes all feeding into a common reservoir, as shown in Figure 1. Assuming that the static pressure across the entire wake is constant and equal to the edge value p_e , the wake velocity profile $u(y)$ will produce a y -dependent total pressure seen by the tubes:

$$p_o(y) = \frac{1}{2}\rho u^2(y) + p_e \quad (1)$$

$$= \frac{1}{2}\rho u^2(y) + p_{o\infty} - \frac{1}{2}\rho u_e^2 \quad (2)$$

The non-uniform $p_o(y)$ will produce a mass flow \dot{m}_i in the i 'th tube. Assuming fully-developed Poiseuille flow in each tube, this mass flow is proportional to the driving pressure difference $p_o - p_r$, where p_r is the reservoir pressure to be measured.

$$\dot{m}_i = \frac{\pi}{128} \frac{d^4}{\nu \ell} (p_{oi} - p_r) \quad (3)$$

At steady-state, all the tube mass flows must add up to zero, and if all the tubes have the same diameter d and length ℓ , the pressure differences must add up to zero as well.

$$\sum_{i=1}^N \dot{m}_i = 0 \quad (4)$$

$$\sum_{i=1}^N (p_{oi} - p_r) = 0 \quad (5)$$

Substituting for p_{oi} in terms of the local velocity u_i , and multiplying by the presumed-constant tube spacing Δy , we get

$$\sum_{i=1}^N \left(\frac{1}{2} \rho u_i^2 - \frac{1}{2} \rho u_e^2 + p_{o\infty} - p_r \right) \Delta y = 0 \quad (6)$$

$$\sum_{i=1}^N (\rho u_e^2 - \rho u_i^2) \Delta y = 2(p_{o\infty} - p_r) L \quad (7)$$

where

$$L = \sum_{i=1}^N \Delta y = N \Delta y \quad (8)$$

is simply the height of the N -tube rake as shown in Figure 1.

The lefthand side summation in equation (7) is seen to be a midpoint-rule integration for the momentum+mass defect

$$\rho u_e^2(\theta + \delta^*) \equiv \int (\rho u_e^2 - \rho u^2(y)) dy \quad (9)$$

$$\simeq \sum_i (\rho u_e^2 - \rho u_i^2) \Delta y_i \quad (10)$$

so that with the constant tube spacing Δy , the final result is

$$\rho u_e^2(\theta + \delta^*) \simeq 2(p_{o\infty} - p_r) L \quad (11)$$

$$\text{or} \quad \theta + \delta^* \simeq \frac{p_{o\infty} - p_r}{p_{o\infty} - p_e} L \quad (12)$$

For the most accurate measurement of $\theta + \delta^*$, it is clearly best to reference the reservoir and local-static pressure transducers to the freestream total pressure $p_{o\infty}$, so that $p_{o\infty} - p_r$ and $p_{o\infty} - p_e$ are direct measurements with no bias uncertainties.

2 Near-wake Corrections

Note that the integrating rake does not give the momentum thickness in isolation. If the measurements are to be used to support or validate drag calculations, this causes few problems, since the measured sum $\theta + \delta^*$ can simply be compared with the corresponding sum from the calculation. If the goal is absolute drag measurement, then it will be necessary to estimate the shape parameter $H = \delta^*/\theta$ to deduce the isolated θ . This can be done fairly accurately if only the minimum velocity in the wake is known.

$$U_{\min} = \min \left(\frac{u}{u_e} \right) \quad (13)$$

Measuring this will typically require a separate total pressure tube placed sufficiently close to the wake centerline.

Assuming the minimum velocity is known, a wake velocity profile is then quite closely approximated by the Coles cosine profile

$$\frac{u(y)}{u_e} = U_{\min} + (1 - U_{\min}) \frac{1}{2} \left[1 - \cos\left(\pi \frac{y}{\delta}\right) \right] ; \quad -\delta \leq y \leq \delta \quad (14)$$

where δ is the wake half-thickness. From the definitions of δ^* and θ , we have

$$\delta^* = \int_{-\delta}^{\delta} \left(1 - \frac{u}{u_e} \right) dy = (1 - U_{\min}) \delta \quad (15)$$

$$\theta = \int_{-\delta}^{\delta} \left(\frac{u}{u_e} - \frac{u^2}{u_e^2} \right) dy = (1 - U_{\min}) \delta - \frac{3}{4}(1 - U_{\min})^2 \delta \quad (16)$$

$$H = \frac{1}{1 - \frac{3}{4}(1 - U_{\min})} \quad (17)$$

and the isolated momentum thickness then follows immediately from the measured $\theta + \delta^*$.

$$\theta = \frac{\theta + \delta^*}{H + 1} \quad (18)$$

The above results also apply to moderately assymetric wakes (such as the one sketched in Figure 1) since each wake half above and below the velocity minimum is still accurately represented by the Coles profile.

Both U_{\min} and H quickly approach unity downstream, making their measurement or estimation less and less critical. Ideally, the rake should be moved downstream until it just captures the entire wake, but in practice the wake thickness and location are somewhat uncertain, and some conservatism will be required.

However the shape parameter is estimated, a correction will still need to be applied to the resulting momentum thickness if u_e/u_∞ at the rake is not unity, as is usually the case. The true profile drag/span is simply the ultimate momentum defect

$$D' = \rho u_\infty^2 \theta_\infty \quad (19)$$

where θ_∞ is the momentum thickness very far downstream. The evolution of the momentum thickness in the wake is given by the von Karman integral momentum equation

$$\frac{1}{\theta} \frac{d\theta}{dx} = -(H + 2) \frac{1}{u_e} \frac{du_e}{dx} \quad (20)$$

which can be integrated from the rake position to far downstream if a unique function $H(u_e)$ is assumed. The particular empirical assumption

$$\frac{\ln(u_\infty/u_e)}{H - 1} = \left(\frac{\ln(u_\infty/u_e)}{H - 1} \right)_{\text{rake}} = \text{constant} \quad (21)$$

results in the well-known Squire-Young formula.

$$\theta_\infty = \theta_{\text{rake}} \left(\frac{(u_e)_{\text{rake}}}{u_\infty} \right)^{(H_{\text{rake}} + 5)/2} \quad (22)$$

3 Design Requirements

In light of the assumptions made in the derivation of relation (12), the following requirements must be met for the result to be valid.

1. The flow in the tubes is fully-developed Poiseuille flow.
2. The flow in each tube has negligible dynamic pressure.
3. The static pressure at the tip of each tube is the local total pressure $p_o(y)$.
4. The pressure in the reservoir is uniform.
5. The tubes all have the same length ℓ , diameter d , and spacing Δy .

Requirement 1 is equivalent to stating that the profile-development entrance length for each tube must be very small compared to the tube's length. This gives the requirement

$$\ell \gg \frac{\dot{m}_i}{4\pi\mu} \quad (23)$$

$$\text{or } \ell \gg \frac{d}{16} \frac{\rho u_\infty d}{\mu} \quad (24)$$

which makes the worst-case assumption that $p_{o_\infty} - p_r \simeq \rho u_\infty^2/2$, corresponding to the entire rake being immersed in a massive separated wake. In any case, it should be possible to meet the Poiseuille-flow requirement without too much difficulty by using sufficiently fine tubes. The only drawbacks of excessively-fine tubes are fragility, and possibly an excessive settling time.

Requirements 2 and 3 are essentially equivalent, since negligible dynamic pressure in the tube automatically implies that the static pressure at the tip is nearly the full total pressure. The requirement is

$$u_{\text{mean}} = \frac{\dot{m}_i}{\rho \pi (d/2)^2} \ll u_\infty \quad (25)$$

$$\text{or } \frac{1}{64} \frac{d}{\ell} \frac{u_\infty d}{\nu} \ll 1 \quad (26)$$

which turns out to be essentially the same as Requirement 1.

Requirement 4 means that the dynamic pressures in the reservoir must be negligible compared to the pressure drops across the tubes. With the dynamic pressures in the tubes already being negligible, this additional requirement is met by making the cross-sectional area of the reservoir larger than all the tube areas put together. With D being the reservoir diameter, the requirement is

$$D^2 \gg N d^2 \quad (27)$$

which can be very easily met in practice. On the other hand, an excessively-large reservoir is undesirable, since it will increase the settling time.

A design parameter which still must be selected is the appropriate overall height L . Clearly, this needs to be sufficiently large to capture the entire wake. However, making

it much larger than the wake width is undesirable, since increasing L will proportionately reduce the magnitude of the pressure signal $p_{\infty} - p_r$, giving reduced accuracy. It is perfectly acceptable to build an oversize rake, and cap any tubes which fall well above or below the wake in order to improve accuracy.

The remaining design parameter is the total number of tubes N . This only influences the accuracy of the midpoint summation approximation to the transverse profile integral. Twenty tubes should be sufficient for typical featureless wake profiles. If the wake thickness is expected to be significantly narrower than the rake height L , more tubes may be appropriate.

4 Compressibility Effects

If quantitative results are to be obtained from the integrating rake, corrections to the above relations may be necessary if the freestream Mach number is appreciable. The exact relation for the pitot total pressure is given by any of the standard isentropic formulas, e.g.

$$p_o(y) = p_e \left(1 + \frac{\gamma-1}{2} M^2(y) \right)^{\frac{\gamma}{\gamma-1}} = p_e \left(1 - \frac{u^2(y)}{2h_o(y)} \right)^{-\frac{\gamma}{\gamma-1}} \quad (28)$$

For adiabatic flows at near-unity Prandtl numbers, the stagnation enthalpy can be assumed to be nearly constant across the wake and equal to its freestream value.

$$h_o(y) \simeq h_{\infty} = \frac{1}{\gamma-1} \frac{u_{\infty}^2}{M_{\infty}^2} \left(1 + \frac{\gamma-1}{2} M_{\infty}^2 \right) \quad (29)$$

Combining this with the second isentropic formula above gives

$$p_o(y) = p_{\infty} \frac{p_e}{p_{\infty}} \left[1 + \frac{\gamma-1}{2} M_{\infty}^2 \left(1 - \frac{u^2(y)}{u_{\infty}^2} \right) \right]^{-\frac{\gamma}{\gamma-1}} \quad (30)$$

which is the relation which would be used with a conventional rake to determine the velocity profile from the measured total-pressure profile and the local static pressure p_e .

Just outside the wake, where $u(y) = u_e$, the total pressure is the same as the freestream total pressure, which implies

$$\frac{p_e}{p_{\infty}} = \left[1 + \frac{\gamma-1}{2} M_{\infty}^2 \left(1 - \frac{u_e^2}{u_{\infty}^2} \right) \right]^{\frac{\gamma}{\gamma-1}} \quad (31)$$

This gives an alternative form for the total pressure profile

$$p_o(y) = p_{\infty} \left[\frac{1 + \frac{\gamma-1}{2} M_{\infty}^2 \left(1 - \frac{u_e^2}{u_{\infty}^2} \right)}{1 + \frac{\gamma-1}{2} M_{\infty}^2 \left(1 - \frac{u^2(y)}{u_{\infty}^2} \right)} \right]^{\frac{\gamma}{\gamma-1}} \quad (32)$$

which then replaces the equivalent incompressible form (2). A second-order Taylor series for equation (32) about $M_\infty^2 = 0$ conveniently recovers the incompressible form, but with a first-order Mach number correction.

$$p_o(y) = p_{o\infty} - \frac{1}{2}\rho_\infty(u_e^2 - u^2(y)) \frac{p_{o\infty}}{p_\infty} \left[1 - M_\infty^2 \left(\frac{\gamma-1}{2} \frac{u_\infty^2 - u^2(y)}{u_\infty^2} + \frac{u_e^2 - u^2(y)}{4u_\infty^2} \right) \right] + \mathcal{O}(M_\infty^6) \quad (33)$$

$$p_o(y) \simeq p_{o\infty} - \frac{1}{2}\rho_\infty(u_e^2 - u^2(y)) \frac{p_{o\infty}}{p_\infty} \left[1 + \frac{\gamma-1}{2}M_\infty^2 \left(\frac{u_e^2}{u_\infty^2} - 1 \right) \right] \quad (34)$$

Equation (34) truncates the Taylor series, and also drops terms which are $\mathcal{O}[(u_e^2 - u^2)^2]$, which is justifiable for a typical small-defect wake profile.

An additional effect of compressibility will be to produce a non-uniform kinematic viscosity profile across the wake. This will affect the tube mass flows \dot{m}_i , which are still given by relation (3), but ν must now be replaced by the local kinematic stagnation viscosity $\nu_o(y)$. Assuming that the dynamic viscosity varies as $\mu \sim T^b \sim h^b$ results in

$$\frac{\nu_e}{\nu_o} = \frac{\mu_e \rho_o}{\mu_o \rho_e} = \left(\frac{h_e}{h_o} \right)^b \frac{p_o h_e}{p_e h_o} = \left[\frac{1 + \frac{\gamma-1}{2}M_\infty^2 \left(1 - \frac{u_e^2}{u_\infty^2} \right)}{1 + \frac{\gamma-1}{2}M_\infty^2} \right]^{b+1} \left[\frac{1 + \frac{\gamma-1}{2}M_\infty^2 \left(1 - \frac{u^2(y)}{u_\infty^2} \right)}{1 + \frac{\gamma-1}{2}M_\infty^2} \right]^{-\frac{\gamma}{\gamma-1}} \quad (35)$$

which is suitably approximated by its Taylor series.

$$\frac{\nu_e}{\nu_o(y)} = 1 + \frac{\gamma-1}{2}M_\infty^2 \left[-(b+1) \frac{u_e^2}{u_\infty^2} + \frac{\gamma}{\gamma-1} \frac{u^2(y)}{u_\infty^2} \right] + \mathcal{O}(M_\infty^4) \quad (36)$$

$$\simeq 1 + \frac{\gamma-1}{2}M_\infty^2 \left[\left(\frac{1}{\gamma-1} - b \right) \frac{u_e^2}{u_\infty^2} - \frac{\gamma}{\gamma-1} \frac{u_e^2 - u^2(y)}{u_\infty^2} \right] \quad (37)$$

The viscosity exponent b can be estimated from the accurate Sutherland's viscosity law

$$\mu(T) = \mu_\infty \left(\frac{T}{T_\infty} \right)^{3/2} \frac{T_\infty + T_S}{T + T_S} \quad ; \quad T_S = 110^\circ K \quad (38)$$

by matching its logarithmic derivative at the freestream condition.

$$b = \frac{T}{\mu} \frac{d\mu}{dT} \Big|_\infty = \frac{3}{2} - \frac{1}{1 + T_S/T_\infty} \quad (39)$$

The rake tube mass flow relation (3) now becomes

$$\dot{m}_i = \frac{\pi}{128} \frac{d^4}{\nu_e \ell} \frac{\nu_e}{\nu_{oi}} (p_{oi} - p_r) \quad (40)$$

and the zero net mass flow condition (5) is

$$\sum_{i=1}^N \frac{\nu_e}{\nu_{oi}} (p_{oi} - p_r) = 0 \quad (41)$$

$$\text{or} \quad \sum_{i=1}^N \frac{\nu_e}{\nu_{oi}} (p_{o\infty} - p_{oi}) \Delta y = (p_{o\infty} - p_r) \sum_{i=1}^N \frac{\nu_e}{\nu_{oi}} \Delta y \quad (42)$$

The summation on the righthand side produces

$$\sum_{i=1}^N \frac{\nu_e}{\nu_{oi}} \Delta y \simeq L + \frac{\gamma-1}{2} M_\infty^2 \frac{u_e^2}{u_\infty^2} \left[\left(\frac{1}{\gamma-1} - b \right) L - \frac{\gamma}{\gamma-1} (\theta_k + \delta_k^*) \right] \quad (43)$$

where θ_k and δ_k^* are the kinematic thicknesses defined with the density profile omitted.

$$\theta_k + \delta_k^* = \int \left(1 - \frac{u^2}{u_e^2} \right) dy \quad (44)$$

Substituting the total pressure approximation (34) into equation (42) produces

$$\sum_{i=1}^N \frac{\nu_e}{\nu_{oi}} \rho_\infty (u_e^2 - u_i^2) \frac{p_{o\infty}}{p_\infty} \left[1 + \frac{\gamma-1}{2} M_\infty^2 \left(\frac{u_e^2}{u_\infty^2} - 1 \right) \right] \Delta y = 2(p_{o\infty} - p_r) \sum_{i=1}^N \frac{\nu_e}{\nu_{oi}} \Delta y \quad (45)$$

which then reduces to the compressible equivalent of equation (12)

$$\begin{aligned} & (\theta_k + \delta_k^*) \left\{ 1 + \frac{\gamma-1}{2} M_\infty^2 \left[\frac{u_e^2}{u_\infty^2} \left(\frac{\gamma}{\gamma-1} - b \right) - 1 \right] \right\} \\ &= 2 \frac{p_{o\infty} - p_r}{p_{o\infty}} \frac{u_\infty^2}{u_e^2} \frac{p_\infty}{\rho_\infty u_\infty^2} \left\{ L + \frac{\gamma-1}{2} M_\infty^2 \frac{u_e^2}{u_\infty^2} \left[\left(\frac{1}{\gamma-1} - b \right) L - \frac{\gamma}{\gamma-1} (\theta_k + \delta_k^*) \right] \right\} \end{aligned} \quad (46)$$

where the higher powers of M_∞^2 and $(u_e^2 - u^2)^2$ have been neglected as before. An explicit if somewhat imposing expression for the kinematic momentum+displacement thickness then follows.

$$\theta_k + \delta_k^* = 2 \frac{p_{o\infty} - p_r}{p_{o\infty}} \frac{u_\infty^2}{u_e^2} \frac{p_\infty}{\rho_\infty u_\infty^2} L \frac{1 + \frac{\gamma-1}{2} M_\infty^2 \frac{u_e^2}{u_\infty^2} \left(\frac{1}{\gamma-1} - b \right)}{1 + \frac{\gamma-1}{2} M_\infty^2 \left[\frac{u_e^2}{u_\infty^2} \left(\frac{\gamma}{\gamma-1} - b \right) - 1 \right] + \frac{p_{o\infty} - p_r}{p_{o\infty}}} \quad (47)$$

Again, the natural reference pressure for the reservoir transducer is clearly $p_{o\infty}$. The ratio of static/dynamic pressure in the equation above can of course be replaced by the Mach number

$$\frac{p_\infty}{\rho_\infty u_\infty^2} = \frac{1}{\gamma M_\infty^2} \quad (48)$$

which is itself obtained from the total/static pressure ratio. The local edge/freestream velocity ratio follows from equation (31)

$$\frac{u_e^2}{u_\infty^2} = 1 - \frac{2}{(\gamma-1)M_\infty^2} \left[\left(\frac{p_e}{p_\infty} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \quad (49)$$

Appendix D: Journal of Aircraft Paper

Appendix A: Record of technical exchange via email

- Mark

Return-Path: drela
 To: James Murray <jmurray@rascal.gsfc.nasa.gov>
 Cc: drela
 Subject: Re: APEX program experiment definition
 In-Reply-To: Your message of Tue, 01 Feb 24 13:07:41 -0500.
 Date: Tue, 01 Feb 24 16:00:04 EST
 From: drela

jim,

If the balancer can go as high as you want, then the low wing leading and high Mach are not necessarily incompatible -- you just start higher. This will also give a lower Reynolds number. Also, having a higher wing leading will not likely increase or signer wing loading will not likely increase or signer the chances of a successful pullout. Again, all that is affected is the altitude at which you pull out. The Mach number right after pullout will be largely unaffected. The main effect of wing loading will be on Reynolds number after pullout.

As far as the minimum Mach and maximum Reynolds numbers and CL needed for validation... this depends on the airfoil among other things. Also, you can't separate the target M, Re, and CL this way. The target is really a surface in M-Re-CL space. Do you have a target airfoil and M-Re-CL baseline to start from? If so, I can tell you which way to go.

You ask about the CL range to test. The reality is that you don't have much say about the CL near the airfoil's leading conditions. You have to slow down right up to stall, otherwise you get a shock buffet. Either way, the drag goes way up.

I cannot tell whether the "glove" you are referring to is just an instrumented section of the wing, or a different airfoil section slid over the wing. If you have a "different airfoil" glove, you want to match the glove's circulation chord x CL with the neighboring wing. If not, then you will shed vortices from the glove ends and possibly get significant three-dimensionality. Frankly, I wouldn't trust data where the circulation was mismatched by more than 2%, say. Note that if the glove has a bigger chord, then its CL should be LOWER than that of the wing. You mentioned the possibility of using a locally high CL on the glove and moderate CL on the wing. If the glove has a bigger chord, this is a bad idea.

Ideally, the glove has the same chord as the wing, in which case you will match the circulation for any alpha if the zero-lift lines of the wing and glove airfoils line up also.

Return-Path: drela
 To: Robert Geenen <geenen@sl.difrl.nasa.gov>
 Cc: drela, diesel
 Subject: Re: FAK Wind Tunnel Data
 In-Reply-To: Your message of Fri, 30 Sep 24 14:53:42 -0500.
 Date: Fri, 30 Sep 24 16:17:07 -0500
 From: drela

Bob,

We got your data e-mailing -- thanks. I think I'll need the documentation being sent by snail-mail to figure out what it all means.

Re: Airfoil work.

The approach we're taking right now is we're looking at a number of various airfoil design concepts. You never really start "from scratch", since airfoil design is always iterative at some level. I've always had good success in "tweaking" an existing airfoil to a slightly different Mach and/or Reynolds number, but it may be that the Kennell airfoil might be targeted too far from the Re-Mach combinations we're talking about. I'll have to sift through the data to be sure, though.

Re: NS solvers.

The problem with all the present NS solvers I'm aware of is that they do not treat the separation bubbles in an adequate manner. Some sort of amplification-based transition criterion is a must, since TS waves dominate separation bubble behavior, and separation bubbles barriers to a proper treatment of separation bubbles in an NS solver, it's just that no one has done it as far as I know. If done right, NS predictions could be quite useful, I think.

The stuff that Eini and Maughmer do addresses the TS transition mechanism reasonably well, but as far as I know, they rely strictly on panel methods and do not deal with transonic flows. Maybe that has changed recently.

- Mark

Return-Path: drela
 To: Robert Geenen <geenen@sl.difrl.nasa.gov>
 Cc: mark@wilbur.difrl.nasa.gov, diesel, drela
 Subject: Re: FAK Wind Tunnel Data
 In-Reply-To: Your message of Thu, 05 Oct 24 15:58:52 -0500.
 Date: <2415550000.AAI347@sl.difrl.nasa.gov>

Date: Sun, 5 Jun 1994 01:02:42 EST
 From: drela

To: Bob

Re: XSES for thick airfoils.
 At issue being equal, the thickness of the airfoil is not an issue for XSES accuracy. I've done calculations of various thick strut fairing sections with reasonable results.

Re: advantages of thicker airfoils.
 I've checked myself at thickness-weight tradeoffs for Langström's new long-range Thessus airplane. For a minimum gross takeoff weight, it wanted a very thick (over 2%) airfoil, which would allow a greater span, whilst allowing less takeoff fuel for a given range requirement. The optimum was extremely flat, though, and the aircraft corresponding to a 15% thick section was just a bit heavier, but also smaller and more practical in that sense.

I expect the tradeoff to be somewhat different for a short range, very high altitude mission like that of Perseus A.

The fuel weight doesn't come into it as much, so the advantages of large span are different in nature - lower max power rather than lower cruise efficiency.

I'm not sure yet what effect this has on the "optimum" airfoil. In general, smaller airplanes want thinner airfoils, since the structural constraints are less severe, and the Reynolds number problems are more severe.

Return-Path: drela
 To: Robert Geenen <geenen@sl.dfifl.nasa.gov>
 Cc: murray@ra.mit.edu, Alex Simagin <dfifl.nasa.gov>
 Subject: Re: AFEX and Related Questions
 In-Reply-To: Your message of Wed, 23 May 24:13:12 -0400,
 <>341123211C.An15259est.dfifl.nasa.gov>
 Date: Tue, 23 May 24:00:13+03 EST
 From: drela

Bob,

Here are a few thoughts on the points of your last email.

1) Re: Tail airfoil. I would go with something like a NASA 3110, 1110, or 2410. These work well down to $Re = 10000$ with no special treatment. A minor redesign can make them effective down to $Re = 50000$. You easily can count on $-0.5 < CL + 0.5$ from the 3110, with this CL range shifted up or down with the cambered 1110 or 2410.

I don't like to rely on trips or vortex generators, since they are a pain in the neck, and involve some risk in sizing, spacing, and positioning. If they work at low altitudes, it doesn't mean that they will work up high, so there is no way to test them.

In order to do this some sort of lift-augmentation system is necessary to lower the wing loading required during the pull-out.

² If you size the tail so that you don't need to exceed $CL = +0.5$ on the tail, then separation.

There is another option which is dependent on the data acquisition system, of some sort of high-speed data acquisition system is employed, data can be collected during the pull-out. The aircraft could be designed for the pull-out maneuver and the data collection runs could consist of a series of dives and recoveries. These runs would consist of data collection at or near C_{max} for various mach and Reynolds numbers as the altitude decreases.

This option is based upon the assumption that a data acquisition system can take "snapshots" of quasi-steady state conditions during the pull-out. I have no idea of what types of systems are available to perform this task.

A CL of 1.0 at $M = 1.25$ is a good design maximum for for flight, and higher than that in terms of M_{crit} , and it becomes harder to ensure a safe recovery.

TW

=====
 Return-Path: drela
 To: Robert Geenen <geenen@sl.dfifl.nasa.gov>
 Cc: murray@ra.mit.edu, Alex Simagin <dfifl.nasa.gov>
 Subject: Re: AFEX and Related Questions
 In-Reply-To: Your message of Wed, 23 May 24:13:12 -0400,
 <>341123211C.An15259est.dfifl.nasa.gov>
 Date: Tue, 23 May 24:00:13+03 EST
 From: drela

The main concern is that if the airfoil is designed to have enough margin to survive the pull-out maneuver the rest of the flight will probably be well within the "safe" operating range. Say you perform a 2g pullout at $CL = 1.2$, $M = 0.6$, when you level off the CL will have dropped to ≈ 0.5 for the same mach number. I dont think these conditions would present valuable data.

I would think that it would be more interesting to begin the flight near the upper performance limit predicted by XSES and probe around in this region. I think that an objective should be to gather as much data as above this predicted performance limit while the aircraft is near 30' 10" ft. As the altitude drops and the atmospheric conditions become normal the data gathered there will cover the "safe" operating range.

Tue Jun 4 10:57:25 EDT 1996

p.5

/usr/users/drela/tex/props/apex/email.all

is not something to worry about. Incidentally, I've heard that's rarely exceeded $CL = +0.2$ in normal operation, but they can go to maybe +0.5 in some violent pitch maneuvers. A simple approach would be to just make the tail volume maybe 25% bigger than some typical small sailplane.

For the ailerons, I would go with strongly asymmetric deflection, maybe +10°, or even 15°. With the airfoil barely hovering in the pullout, any significant downward aileron deflection blows off the boundary layer and nixes any lift addition. Upward deflection is always effective, and produces yaw moment in the right direction.

It might be very productive to build an RC glider model of the APEX to figure out suitable control throws, CG limits, trim settings, adverse yaw, etc. There should be some local RC flyer, inside or outside of NASA, who could drool at building something like this. To be conservative, I would size it to give Re maybe 20% smaller than the real APEX at altitude. Even with no Mach effects, you'll learn quite a bit from it, I think.

$3 \times S = 4 \text{ ft}^2, 3\text{km}$ gives

$M*sqrt(CL) =$	1.48	1.59	2.3
$Re*sqrt(CL) =$	245K	245K	280K

transonic effects start to set in at 1.53, $M*sqrt(CL) \sim 3.6$. The airfoil will die above $M*sqrt(CL) = 3.67$ or so, so this is the target range. At the same time, I would like to see $Re*sqrt(CL) < 200K$ if possible. Clearly, for the 3.1 ft chord, $W/S = 4.0$ is desirable. Higher W/S will tend to increase Re rapidly, not just because of the higher flight speed, but because you will finish the pullout at a lower altitude. Have you done the simulation runs to ascertain the effect of W/S on minimum Re? Maybe going to higher aspect ratio may be a way to get lower Re with a reasonable wing loading. Higher AR will also mean smaller energy loss in the pullout and any high-g turns which may be necessary to drive up $M*sqrt(CL)$ during data acquisition. On the other hand, a higher AR adds weight. I haven't done the numbers to see if the higher AR buys you much overall.

Incidentally, Tom's latest airfoil looks extremely attractive. It's quite different from the Perseus section, which does not appear to be at all suitable for these Reynolds numbers.

4) The Perseus airfoil was designed for well-subsonic operation $W = 5.35$ at 2 km. Since then, the installed power and wing loading have more than doubled! Now, the

wing airfoil will start to run into Mach trouble at around 2.5 km, which is actually a reasonable margin from 2.5 km, it can't go much above $M*sqrt(CL) = 3.67$, since it is way overcambered for high-Mach operation. Still, I don't expect this to be a limiting factor for Perseus A. It looks like the things that are more likely to limit Perseus are the Bregt available excess power, and/or low capacity -- all directly caused by the weight growth.

5) I have a student building a model of the front-mounted radiator airfoil. We hope to be in the tunnel early next year.

Mark

=====

Return-Path: drela
 Date: Fri, 9 Dec 1994 17:23:32 -0500
 From: diesel@Thomas.Washington.CC
 To: drela, geenen@slc1.nas.nasa.gov

what I have looks to be a good baseline airfoil for this mission. It can be used "as is," or with small modifications (camber, thickness...), to adapt to the evolving aircraft.

The section is 13% thick, with about 4% camber. It was designed for the pull-out recovery, $CL = 1.2-1.3$, $W = 5.5-6.6$ or so... .

The "incompressible" C_{lmax} is around 1.36. This limit is lowered significantly with increasing Mach number to around $M*sqrt(CL) = 5.65$, where the boundary layer is blown off the top by the shocks. Its pretty safe to say the upper limit is still $M*sqrt(CL) = 6.65$.

The lower end is up to debate. Presently, the leading edge separates on the lower side at around $CL = 5.5$. This can be adjusted depending on aircraft needs.

Unless there are any other requests, this is the airfoil I will be using to collect the data. Going beyond the point-of-no-return should be no problem. Again, just let me know where you need data, and in what form and I can get things running.

Tw

=====

Return-Path: drela
 To: Robert Geenen <geenen@slc1.nas.nasa.gov>
 Cc: diesel, drela
 Subject: Re: APEX Airfoil
 In-Reply-To: Your message of Thu, 15 Dec 94 14:15:56 -0500.
 Date: Wed, 21 Dec 94 00:34:52 EST
 From: drela

Bob,
 Tom left for the holidays just after you sent your last message.

I put the APEX 1.4 airfoil which Tom sent through a few optimization cycles, making the APEX 1.6, which is a slight improvement all around. I'd like to make this the "official" current airfoil. Overall, it is quite well-behaved up to Mach .65, or M=0.65. It begins to fail's apart after that.

I ran out almost all the M,Re combinations you requested.
 I'll send a untarred, tar file of the directory which contains all of these Polar files. Unpack as before:

```
* ungzcode mail-file
* it's NOT necessary to remove email lines
* ungzipress apelix.tar
* tar xf apelix.tar
```

The file naming is Ma-Re .65.1600 is a M=.65, Re=1600 Polar. You can plot any combination of Polars using the IRIS executable "gplot" which is in the directory. Just run it -- it's menu-driven. If you are not on an IRIS, I can send the source and plot library for Eplot.

I've also included some random *-box PostScript files. Some of these you can regenerate as needed with Eplot. The coordinates are in the blade.apelix file.

You'll notice that I ran M=.75 Polars instead of the M=.65 you requested. At M=.80, the airfoil generally refuses to converge, indicating large-scale unsteadiness. Mach buffet. Looking at the CL-CD Polar trend, it also looks like the polar would collapse to near-nothing at M=.80. For the CL13, it seems this would happen at a somewhat higher Mach, like 0.85. In case you want to compare these results with the tunnel data you sent, keep in mind that those tests had significant 3D relief from the rather higher low aspect ratio, which reduces the effective Mach number. The higher Mach also causes the effective aspect ratio itself to be reduced, according to the Prandtl-Glauert rule. At M=.8, the effective AR is 3.6 times the actual AR. The 3D relief effect will not be very significant on the high-AR Apex aircraft, so the 2D Polars are much more representative.

You also asked for post-stall Polars. Again, I can't get steady solutions once the drag starts to skyrocket. If you need data for higher alphas, I would just extend the last CL to whatever alpha you need. This should fall within the uncertainty and/or unsteadiness band.

- Mark

=====
 Bob,

Return-Path: drela
 To: Robert Geenen <geenen@sci.dirf.nasa.gov>
 Cc: drela, diesel, Alex.Simonegate.mit.edu, bowers@willbur.mit.edu,
 Elise_Gravance@mgate.mit.edu
 Subject: Re: APEX 1.6 - Cd
 In-Reply-To: Your message of Mon, 23 Jan 25 10:16:37 -0800.
 <9501292015.AAC65sci.dirf.nasa.gov>
 Date: Mon, 23 Jan 25 10:30:12 EST
 From: drela

Date: Wed, 21 Dec 24 03:50:55 EST
 From: drela

The "undervent" on the front lower surface of APEX 1.6 is just to add leading there -- it's sort of like bottom aft camber but at the front. It increases Clmax and reduces Cm [1]. Despite these attractive features, you don't often see this on airfoils since it kills the low-CL end of the polar, but for a HAEL application the low-CL end is irrelevant.

If you just use an untapered flat wing, you will get very benign stall characteristics with no special treatment. Just keep in mind that the average CLmax will decrease, since there will be some local CL decrease towards the tip. How much depends on the effective aspect ratio (corrected for Mach number with Prandtl-Glauert). A single lifting-line solution should give you the local CL pretty accurately.

The roughness waviness is much less significant than on a sailplane -- that's one nice thing about low Reynolds numbers. If you are using sailplane-level technology, like female molds, etc., then waviness is a non-issue.

- Mark

=====
 Return-Path: drela

To: Robert Geenen <geenen@sci.dirf.nasa.gov>
 Cc: drela, diesel, Alex.Simonegate.mit.edu, bowers@willbur.mit.edu,
 Elise_Gravance@mgate.mit.edu
 Subject: Re: APEX 1.6 - Cd
 In-Reply-To: Your message of Mon, 23 Jan 25 10:16:37 -0800.
 <9501292015.AAC65sci.dirf.nasa.gov>
 Date: Mon, 23 Jan 25 10:30:12 EST
 From: drela

The positive d α m delta is typical for low-Re airfoils, and the transonic flow makes it worse. There's no good way to alleviate this that I'm aware of. Yes, the effect is much smaller at higher Re. You won't see these effects

with the Egpler-Somers idea, since they do not account for the viscous displacement effect on the pressure distribution - at least that's my understanding. This viscous displacement is the mechanism responsible for the Δc_m behavior. MSES does duplicate the Δc_m behavior seen in Abbott and Doenhoff, for example.

You actually identified a very good point:

Experimentally determining Δc_m behavior in the high-M, low-Re regime is something that perhaps should be one of the details of APEX. In depends on Δc_p details, which may not be captured well enough in the calculations. Certainly, there is no reason to expect that the 2m behavior will mimic conventional high-M, high-Re transonic airfoils, since the shock structures are vastly different. Clearly, this is something you want to measure. However, this will require Δc_p measurements at a fairly large number of points - much like pressure-tapped wind tunnel airfoil model.

- Mark
 returnPath: drela
 To: Robert Seeren <geenen@sl.dfrr.nasa.gov>
 cc: drela, dieser, bowers@ilcifl.dfrr.nasa.gov
 Subject: Re: APEX - Update Questions
 Ir-Reply-To: Your message of wed, 21 feb 25 10:42:47 -0800.
 Date: wed, 21 feb 25 15:21:20 est
 From: drela
 Fcc:

We haven't had much rain OR snow here. Both Tom and I are cross-country skiers, and we're Pissed! Last winter (34" total snowfall) was like paradise. But I digress....

>1. What's the best way for us to get a copy of MSES? I have an old version of ISES right now, so it's probably time to upgrade.

I can easily send it in one big tar file. If you have Dec, IRIS, HP-9000, or RS-6000 workstations, I expect it to run right out of the box. Sums are almost as easy; who should I send it to?

Incidentally, the migrate addresses in the cc list always bounce for some reason:

Alex_Sim@atmgate.dfrr.nasa.gov
 Elise_Gravance@dfrr.nasa.gov

>2. Is it worth the effort to model our control surfaces using MSES? Have you encountered unique problems with deflected control surfaces for the low Reynolds number, high Mach case?

Yes, I expect there will be a very large nonlinearity for positive aileron down deflections. You want gates large aileron deflection assumption, up down = 3% or more. Near CLmax, the aileron adds mostly drag and hence adverse Δc_m , so there's no point in deflecting it down more than a few degrees. The up-aileron is much more effective.

>3. Where is the separation point with respect to the transition point? Can we get plots of separation location? I know that the idea is to have transition occur as soon after separation as possible to increase the chances of reattachment. Is that what happens on the APEX airfoil?

At high lift at low Re, there are typically two separation locations:
 - laminar separation at the start of the bubble
 - turbulent separation near the trailing edge

I don't print these out since they don't tell you very much. A much more relevant quantity is the kinematic shape parameter R_K distribution. The peak value in the separation bubble indicates the magnitude of bubble losses, and the peak value at the trailing edge indicates how close you are to separation, or how "bad" the separation is. The MSES Plotter MPLOT gives a plot of R_K vs. x.

>4. Back before Christmas I asked about surface roughness/waviness criteria. Your reply made it sound like low Reynolds number airfoils are not sensitive to surface waviness or roughness. That's seems to go against what I know about transition triggering mechanisms. Could you elaborate? Part of the reason for this discussion is that we are already talking to wing designers and builders and they are asking what's realistic; they will need to ensure I've been telling them that typical sailplane construction techniques will be adequate, but I need more detailed info on this topic.

I didn't mean to say that roughness/waviness is not important for APEX -- just that it's significantly LESS important than on a sailplane. In fact, at the lowest Re = 1000, say a little bit of strategically-placed roughness or waviness can help, but the appropriate location changes with CL, which is one of the reasons I don't like to rely on BL trips.

I understand that the builders want a definite number for the maximum deviation, but the deviation by itself is meaningless aerodynamically. You also need the wavelength of the deviation. For example, adding a 1/16" bump to the surface everywhere will have practically no effect. On the other hand, adding a 1/2" high, 1/2" long bump on the upper surface at 5% chord would be a disaster. The location itself is just as important. That same 1/16" high, 1/2" long bump placed on the bottom surface at 25% chord would be totally innocuous.

There are two approaches you can take here:

a) Expend enormous efforts at quantifying the effects of inaccuracy;

magnitude, xavelx3t, and zlation, and then tell the builders just what accuracy is needed at each surface location.

e. use `workxx`.

I vote for approach b., using the sailplane-level of quality; as an appropriate level of general. If there is a significant cost quality tradeoff for the models, I would tell the builders to sink more money into the front 50% of the upper surface and around the leading edge. The rest of the airfoil is much less important.

> I was wondering how sensitive MES is to smoothness of coordinates?

> Could the effect of surface waviness on airfoil performance be determined by adding waviness to the coordinates given to MES?

MES can predict the effects of long-wavelength deviations, but not for short ones. Very short waves are in effect roughness. I don't know what the lower limit for the "trusted" wavelength is, so I wouldn't trust any such study. Part of the problem is that for efficiency reasons, MES uses the ein "envelope method", which is somewhat simplified form of the full-blown `env` transition prediction methodology. The envelope method assumes that the BL evolves in a reasonably gradual manner. The strong periodic whacks it would get from intentional surface waves might put its accuracy into question. I've always worried about this, so a check, I set up KPLCT to show the transition location you could have gotten from the full-blown `env` method.

> The last question I had was about the design philosophy of the APEX airfoil. With no airfoil design experience I was wondering what trade-off if any was made in order to ensure the wide, flat drag bucket? Is there any possibility that drag could be sacrificed to get better Cm characteristics? I think this is a long shot, but it never hurts to ask.

Tom might want to add to this, but as I see it, the key problem was controlling the separation bubbles over the operating range of Mach numbers. We could have done better at the higher Mach by having a flatter "rooted" CP there, but this causes an LE spike which kills the upper surface BL at lower Mach. A compromise resulted the airfoil being "OK" at all Mach numbers, but not the best that you could do at any specific Mach number. This is typical of any compromise design, of course. I used the optimizer at the end just to fine-tune this compromise.

As a separate issue, the reliance on bottom aft loading had to be reduced, since at low Re the bottom BL couldn't negotiate the high bottom aft pressure that the Keenally section has, for example. Instead, we relied more on front loading, which doesn't have the low- Re BL separation problem at all. You can't get as much lift out of the front loading as you can jet out of aft loading, but on the other hand, the front loading permits a nearly all-laminar bottom surface, which is good news for CL.

- Mark

```
Return-Path: drela
Received: from localhost by henry (128.125.1.5.2) via TCP/5-2535PM
18 Apr 1996 Sat, 20 Apr 12:55:22-0400
Message-ID: <210105125522.442@henry>
To: Robert.Beehan <robert.beehan@atnf.csiro.au>
Cc: drela, alessio_bowers@silbury.atnf.csiro.au
Subject: Heat Exchangers
In-Reply-To: Your message of "Wed, 21 Feb 25:15:21+0200 EST."
Date: Sat, 20 Apr 26:20:21+0400
From: drela
X-Msg: SMEF
```

Bob,

I'll send you a more-or-less final version of the Heat-Exchanger paper, in uuencoded, compressed form.

The return file from the wind-tunnel test you guys paid for is that the front-mounted HV installation looks quite promising. The data is kind of ragged, since the wind tunnel model was not very accurate undergraduate + team + fibreglass + ME, no-machining . Also, the instrumentation was rather low grade, and tunnel time was too short to permit a thorough checkout of spanwise uniformity, flow measurement, etc. I'll also send a plot of the design geometry and what was taken off the actual model to give you an idea of the deviation in geometry. It looks awful, but the model was fairly uniform spanwise. I ran the calculations with the model geometry, so the results are valid for comparison purposes. Incidentally, the test model geometry is computed to have a significantly lower C_{Lmax} -- by about 10%.

The encouraging news is that the measured drag of the airfoil HV model is astoundingly low given the very large radiator area it's carrying, even at the rather low Reynolds number.

The important result from this test is that a low velocity ratio of $|V|_{infty} = 0.10$ is possible at 35 km altitude, which means that getting 0.25 or less down to $|V|_{infty} = 0.35$ at 10 km is absolutely essential if you want to fly significantly above 25 km. Otherwise, the radiator drag will consume most of your engine power at ceiling, see Figure 3 in the paper.

This test might suggest some "real" followup tests. One is to see how small a 3D wing can you get versus Reynolds number. We couldn't get above $Re = 450K$ in our wind-tunnel without driving the motor a hernia.

It would be particularly useful to see how small you can make it at $Re = 2.5M$ - 15 km, since this is a key sizing parameter. Also, having an accurate model and good instrumentation couldn't hurt either.

- Park
- Avg CL = 0.78 $\epsilon = 0.32$
- Assuming $L_{max} = 1.2$, this means you are throwing away 1 - 0.78 = 22% of the airfoil's maximum lift capability! There is also the problem of the changing radius across the test section, which will produce significant 3-D effects on the potential flow and possibly on the boundary layers.
- A simple flat wing has the following CL variation at $M = 5.5$:
- | CL | flat wing |
|-----|-----------|
| 0.0 | 1.25 |
| 0.1 | 1.16 |
| 0.2 | 1.10 |
| 0.3 | 1.05 |
| 0.4 | 1.02 |
| 0.5 | 1.00 |
| 0.6 | 0.98 |
| 0.7 | 0.96 |
| 0.8 | 0.94 |
| 0.9 | 0.92 |
| 1.0 | 0.90 |
- Avg CL = 1.04 $\epsilon = 0.265$
- This costs you only 1 - 1.04/1.2 = 13% of L_{max} , and the CL variation at the test section is more uniform but still not ideal. Note that this has *less* TI than the 10 deg washout wing, which is very over-twisted.
- I would favor a small washout for reasons 2 and 3 above. The following washin distribution looks very good from a strictly aerodynamic viewpoint:
- | z/ft | alpha_deg |
|------|-----------|
| 0.0 | 0.0 |
| 1.0 | 0.8 |
| 1.5 | 1.6 |
| 1.8 | 2.0 |
| 2.0 | 4.0 |
- It gives the following CL variation:
- | CL | +4 gradual washin |
|-----|-------------------|
| 0.0 | 1.25 |
| 0.4 | 1.25 |
| 0.7 | 1.17 |
| 0.8 | 1.11 |
| 0.9 | 0.95 |
| 1.0 | 0.90 |
- Avg CL = 1.12 $\epsilon = 0.206$
- This gives the most lift, but more important is the nearly-flat CL at midspan. D_L is bigger, but the penalty is tiny compared to the total drag. Also, CL is lower at the tips despite the washin. Note also that the washin wants to be sharply nonlinear. A linear washin gives the following:
- | CL | +4 linear washin |
|-----|------------------|
| 0.0 | 1.11 |
| 1.0 | 0.95 |

卷之三

This has a TL peak at midspan rather than at the root, which may not be too bad.

My feeling is to go for the washin, preferably the individual one, with maybe a bit less than the full 18 degrees to keep the stability & control folks from gasping. The flat wing is OK if you don't want to make a left and right wing mold, but it's not ideal from the aspect measurement viewpoint. Washout serves no purpose that I can see.

If you go for left and right molds, you might as well taper the planform a bit, from 70% span outward, say, with modest taper, zero twist looks appropriate, so the whole washout thing may be a non-issue. Also, taper would also greatly alleviate aerelasticity, which may or may not be a problem.

Re: Aileron effectiveness. I would use strongly what I mentioned this before. I would use strongly asymmetric throws, like +15 or even 30°. Tom is looking at the effects of aileron deflection to quantify this better.

•

I don't think that tail. It flap deflection trait for a tail a bit I'm certain the especially with a numbers.

Looking at the APEX PER document, it struck me how small the tail aspect ratio is. Someone obviously wanted to keep the tail Re high, but that seems a bit extreme. I would think that a somewhat higher tail aspect ratio would have less tail area, and hence less weight for the same dCn/dalpha. R=200 or even 150K doesn't seem like a lot if the CL doesn't exceed 1.5, say. I really don't know what the tail CL limits are?

Re: Reporting, [http://www.fcc.gov/encyclopedia/](#) in the spirit

of "reports", and is a lot more time-efficient. I am saying it all and can put it in a hand-bound volume for the bureaucrats at both ends if they want the turned data in the heat-exchanger paper as a breeches-receipt of sorts.

I certainly plan to put together a final document of some sort. I wasn't aware that the progress reports go to CASI. I thought that it was just the final report. Frankly, I think that the progress reports would just fill up their file space, since it would all get relegated in the final report anyway. I can certainly organize and send them that interim stuff if you raise a stink, but otherwise, it seems like a waste of everyone's time.

4 degrees to keep the stability & control folks from gasping. The flat wing is OK if you don't want to make a left and right wing mold, but it's not ideal from the aero measurement viewpoint. Washout serves no purpose that I can see.

If you go for left and right molds, you might as well taper the planform a bit, from 75% span outward, say, with modest taper, zero twist lobe appropriate, so the whole washout thing may be a non-issue. Also, you could also greatly alleviate aerelastic excitation, which may or may not be a problem.

* * *

Re: Aileron effectiveness.
I think I mentioned this before. I would use strongly asymmetric thracs, like +1-5 or even 2-5. Tom is looking at the effects of aileron deflection to quantify this better.

Re: Tail airfoil.
 I don't think that the APEX 16 is appropriate for the tail. It is very intolerant of downward flap deflection (up elevator), which is a 'very' bad trait for a tail airfoil. Tom can check this out, but I'm certain that a TACA XX10 would be much better especially with a little tuning for the low Reynolds numbers.

It turns out that the transonic effects at the ends of the polar have roughly the same effect on $C_{p,x}$ as moving the max-thickness point forward at low Mach, so that these airfoils look well-matched to the lower Re at the higher Mach. In fact, the transonic suction side $C_{p,x}$ for the 2416 is quite similar to the APEX 16, but looks to be tailored for somewhat lower Re. If you look at the polar, the 1410 and 2416 are doing fine at $Re=1500$, while the APEX 16 starts to seriously drop out at $Re=2000$. Clearly, they are better than the APEX 16 for the tail.

Looking at the bellars of the hills and \$115. I think you
are 16 for the call.

want something midway, i.e., a $\Delta x = 1.5$ 412. This should give you a useable CL range of $-0.4 \leq C_L \leq 0.5$, which hopefully meets the requirements. Note that the APEX is won't go much below $C_L = 0$. The bottom fence needs to be only about $x = 1.15$. The top fence needs to be only about $x = 1.5$, the height of the top fence at most.

Yes, batvies should be on the bottom of the wing, and the top of the tail. I don't see any problems with belltranks striking out on either side, as long as they are not immediately next to the test section.

The only nuisance with the NASA sections is the irregularity in α around $\alpha = 0$, which is due to a large separation bubble sloshing around near the trailing edge and upsetting the Rutta condition. On a 3-D surface this wouldn't be nearly as much of a problem, since the bubble is not likely to sit at the TE all across the span at any given alpha. So in a spanwise-averaged sense, the $\Delta \alpha$ disturbance will be much weaker. Still, it may be a good idea to put a row of turbulator bumps just ahead of the hinge line to kill the bubble and make the elevator behavior more linear and predictable. My aversion to turbulators on them -- they're just cheap insurance, if they trip the BL too early, it's of no consequence, unlike on the wing airfoil. The turbulator bumps want to be quite large -- at least $1/4$ high.

- Mark

Return-Path: drela@mit.edu

Received: from localhost by henry; 15:55 1:1.5.2 Sat Apr 25 0203PM
id AA1255; Tue, 25 Aug 1995 15:13:59 -0400

Message-ID: <355072113.AA1255@henry>

To: Robert Geffen <geffen@osl1.dfrc.nasa.gov>

Cc: drela@mit.edu

Date: Tue, 29 Aug 25 15:17:08 -0400

From: drela

X-Mts: smtp

This is to let you know that I'm back at MIT.
Is there anything you need me to do right now?
I've scanned the APEX Engineering Meeting Summaries to date, so I kind of know what's going on.

Return-Path: drela@mit.edu
Received: from localhost by henry; 15:45 1:1.8.2/96Apr25-0203PM
id AA1255; Wed, 7 Jun 1995 17:13:53 -0400
Message-ID: <355072113.AA1255@henry>
To: Robert Geffen <geffen@osl1.dfrc.nasa.gov>
Cc: drela@henry.mit.edu, drela@henry.mit.edu
Subject: Re: APEX Design
In-Reply-To: Your message of "Wed, 07 Jun 25 12:47:56 EDT."
<>355072113.AA1255@henry>
Date: Wed, 07 Jun 95 17:13:52 -0400
From: drela
X-Mts: smtp

Bob,

The NASA 2412 for the tails is OK with me. A 2410 would go to somewhat lower Re , but that's probably not too much of an issue if you use turbulator bumps ahead of the hinge line, as I discussed earlier.

I'm not so sure about the wing fences. I don't think you want them ahead of mid-chord, since the extra-slow fluid in the fence wing corner will separate before the wing airfoil and possibly suck up the wing's BL at the test station. Very minimal fences between $x = 0.5 \dots 1.0$ should be OK to prevent any separated fluid from the wing root migrating out across the span. My first guess is to start the fence at $x = 0.6$, extend it aft tangent

to the airfoil surface, ending up at about $y/c = 0.55$ at the trailing edge, and extending back to $x = 1.0 \dots 1.15$. The bottom fence needs to be only about $x = 0.5$, the height of the top fence at most.

I'll be at Boeing from June 12 until August 22. I'll be checking my MIT email whenever possible, but I won't have easy direct access to my machine. If there is anything you want me to calculate before I leave, let me know ASAP. Tom is preparing to fly jets for the *AEPEX*, so he won't be around at MIT for the summer either. I'll be back at MIT from September on.

Mark

Return-Path: drela@mit.edu
Received: from localhost by henry; 15:55 1:1.5.2 Sat Apr 25 0203PM
id AA1255; Tue, 25 Aug 1995 15:13:59 -0400
Message-ID: <355072113.AA1255@henry>
To: Robert Geffen <geffen@osl1.dfrc.nasa.gov>
Cc: drela@mit.edu
Date: Tue, 29 Aug 25 15:17:08 -0400
From: drela
X-Mts: smtp

Bob,

This is to let you know that I'm back at MIT. Is there anything you need me to do right now? I've scanned the APEX Engineering Meeting Summaries to date, so I kind of know what's going on.

One thing that I see that needs serious attention is the wake rake setup. I think it is vitally important that measurements be done at various spanwise locations, if only at a few selected operating points. This is to make sure that there is no significant spanwise variation. Ideally, you want to measure the 2D as a spanwise average across a significant portion of the wing -- maybe with a number of rakes, or a translating rake on a leadscreen.

I've seen rapid wake 2D variations as much as $\sim 5\%$ across the span of a supposedly 2-D model setup, so that the averaging can be very important. Note that even such strong spanwise variation doesn't imply that the 2Dness of the flow is strongly compromised. There is very little resistance to the slow BL fluid from migrating slightly along the span, and even slight variations in spanwise migration add up to a lot at some distance downstream.

The total spanwise-averaged 2d is unaffected by this migration, but you can get very misleading results if you measure at only one spanwise point.

Since our last exchange, I've reconsidered the zenses, and now I think that the fences may actually aggravate the spanwise nonuniformity in 2d rather than help. If there is a general tendency for the BL to cross outward overwise, say, it will "pile up" against the outer fence and have a higher measured 2d there. Likewise, it will thin out at the inner fence. If you leave off the fences, you will avoid this piling up and thinning out, and quite likely get a more uniform measured 2d. Is there any flight experience with fences on wing fences? Have they been demonstrated to improve the data? If not, I would leave them off.

- Mark

```
=====
Return-Path: drela
Received: from localhost by henry; 15 Jun 1.1.8.2 06 Apr 95 09:3PM
Id: AA17241; Tue, 29 Aug 1995 21:42:41 -0400
Message-ID: <95083003112-AA199513henry>
To: Robert Geenen <geenen@sls1.dirr.nasa.gov>
Cc: drelahenry.MIT.EDU, bowersarigel.dirr.nasa.gov
Subject: Fences, Rakes, etc.
In-Reply-To: Your message of "Tue, 29 Aug 95 15:30:27 PDT."
<9508292230-AA117783ccs..dirr.nasa.gov>
Date: Tue, 29 Aug 95 21:42:41 -0400
From: drela
X-MUA: smtp
```

Bob,
Instead of the sheet metal fences to hold the rake support, you could just go with thickwall tubes mounted to the bottom of the wing with brackets. The tubes can carry the pressure tubing or wires, depending on where you put the transducers or scanivalves.

The + - 2 inches of rake travel you mention seems too small to me. I would go for at least + - 1 foot, so that you cover at least a chord width. I see two ways to go here:

1) use 1 rake with a traverse mechanism as you suggested.
2) use several fixed rakes.

Option 1 obviously requires less data channels, but option 2 has almost everything else going for it:

- * Mechanical simplicity and reliability. This is obviously important with the small number of APEX flights envisioned. Also, it's hard for me to see how the pressure tubing could allow for the rakes motion without

having a flexed tubing bundle flapping in the breeze and shaking everything.

- * Shorter development and construction time. I've built a number of wake rakes. They are trivial compared to a motor/traverse mechanism. Also, once the milling machine is set up or programmed for building the rake body, building several rakes doesn't take much additional time. Since APEX is behind schedule to gather, the faster you can build the rake system the better.

- * Simultaneous spanwise measurements. This means that you don't have to slew the rake, wait for the pressures to settle, and then make another spanwise measurement. Each operating condition can therefore be shorter, so that you'll be able to do more operating points per flight. It's hard to argue against having data from more operating points from any given flight!

Obviously, I'm in favor of the multiple rakes. I'm also willing to do a rough sizing and mechanical design for the system as you suggested. I would need to know the specs on the pressure transducer/scanivalve system you will be using, number of channels allotted, etc. I would also need the structural X-section of the wing, so that I can figure out how to mount the whole thing.

When is the CDR?

- Mark

```
=====
Return-Path: drelahenry.MIT.EDU
Received: from FAETIC.JARPER-ANNE.MIT.EDU by henry; 3 Jun 1.1.8.2 06 Apr 95 09:3PM
Id: AA17241; Wed, 16 Oct 1995 17:42:50 -0400
Received: from HENRY.MIT.EDU by MIT.EDU with SMTP
Id: AA17248; Wed, 16 Oct 1995 17:42:46 EDT
Received: from localhost by henry; (5.65.1.1.6.2.0)25.220.3PM
Id: AA17211; Wed, 16 Oct 1995 17:42:43 -0400
Message-ID: <9511021404AA17211henry>
To: Al Bowers <bowers@willbur.dirr.nasa.gov>
Cc: drelahenry.EDU
Subject: Re: APEX wake rakes...
In-Reply-To: Your message of "Wed, 18 Oct 95 11:10:34 EDT."
<9511021810-AA1282@willbur.dirr.nasa.gov>
Date: Wed, 18 Oct 95 17:40:43 -0400
From: drelahenry.MIT.EDU
X-Nntp: snmp
```

Al,

I agree with Dan Genners... I don't think that there's any danger of steriler variations forming. However, any spanwise variations are more likely to be caused by the slight 3D-ness of the wing, the pressure ports,

rake mounts, etc. The change of spanwise variation is not large, but it can't be ruled out based on prior high-Re glide experience. The presence of large separation bubbles makes this section E₂₄ more susceptible to significant spanwise-induced perturbations.

In any case, I have an idea: I propose we use one fixed dense rake to measure the wake profile, and add several fixed integrating rakes on either side to check for non-uniformity. As you may know, an integrating rake measures the sum $\delta\text{ta}^* + \theta\text{eta}^*$. It has some uncertainty depending on what you assume for the flow in the pressure tubes and reservoir, but this uncertainty should be the same for all the integrating rakes, so for comparative measurements they should be very good. It is essential that all the integrating rakes are essentially identical, but that shouldn't be a problem. Of course, the big advantage is that an integrating rake requires only one channel.

If this sounds OK to you, I can lay out both the real rake and the integrating rake geometry. What is your current estimate of the number of ports available for all the rakes? This affects how much sweat I have to expend to position the rake tubes properly. Also, have you people ever worked with integrating rakes? If not, I can give a few pointers.

- Mark

```
=====
Return-Path: drela
Received: from localhost by henry; 5.65.1.1.8.2.156APR25-02:33PM
id AA265; Fri, 26 Oct 1995 15:00:36 -0400
Message-Id: <3512062003.AA0025@henry>
To: Al Bowers <owers@milbur.dirn.nasa.gov>
Cc: drelahenry.MIT.EDU
Subject: Re: APEX wake rakes.
```

In-Reply-To: Your message of Fri, 20 Oct 95 02:55:59 PDT.
<3512061555.AA0147@milbur.dirn.nasa.gov>

Date: Fri, 26 Oct 95 16:00:35 -0400

From: drela

X-Mts: smtp

CF, let's go for the integrating rakes, then. I don't have any detailed references on them, but I've analyzed them and it's not too hard to see what is required for them to work as intended:

1. Some of the tubes have flow going in, and some have it going out obviously. For the whole rake to work as expected, the pressure drop across each tube must be proportional to the mass flow through the tube. This means that each tube must have fully-developed laminar Poiseuille flow over most

of its length, with the flow velocity \ll laminar. It is also necessary that the end of each tube sees the full local stagnation pressure. Both of these requirements demand that the tubes be sufficiently long and thin.

2. The length of each tube must be inversely proportional to the total tube spacing. Of course, the simplest thing is to use evenly spaced equal-length tubes.

3. The reservoir into which the tubes dump into must be sufficiently large to have negligible velocities along its length.

If these conditions hold, then the pressure in the wake reservoir is

$$P_{\text{rake}} = P_{\text{static}} + 2.5 \rho h \text{ta}^* + \delta\text{ta}^* + \theta\text{eta}^* - L$$

where $2.5 \rho h \text{ta}^*$ is the local dynamic pressure just outside the wake, and L is the height of the rake.

I would reference all rake scaninave pressures to freestream P_{total} . With a regular or integrating rake wake the thing you're after is deviations from P_{total} , so that this is the logical reference. For example, measuring $P_{\text{rake}} - P_{\text{static}}$ gives you $1 + \delta\text{ta}^* + \theta\text{eta}^* - L$, while measuring $P_{\text{rake}} - P_{\text{total}}$ gives you $(\delta\text{ta}^* + \theta\text{eta}^*)_L$ directly. The latter quantity can be quite small relative to 1, so you want to measure it directly if at all possible.

The wing surface pressures may be best referenced to freestream static, although here it doesn't matter so much, since the pressure deviations are larger. Using P_{total} is always "safer", I think, since it is much easier to measure reliably.

I propose the following setup.

The "master" P_{total} for referencing all scaninave. This cut tap off the air-data probe, although you may want a separate probe to decouple the instrumentation from the aircraft flight-control systems.

One P_{static} referenced to master P_{total} from a trailing probe or whatever to get pitot pressure. This is then slightly corrected for Mach to get true dynamic pressure $0.5 \rho h c^2$.

Another P_{static} referenced to vacuum for pressure-altitude. I assume:

The dense rake should have its own local static probes in addition to the usual total tubes all referenced to the master P_{total} . Ideally, you want one static port above and below the wake.

The integrating rakes have only one pressure value referenced to master p_{total} .
 Inboard thermister placed in a bleeding slot. Frote to detect total and hence density and viscosity.
 This is necessary only for Reynolds number and true airspeed.

The 32 dense takeoffs are barely enough, but should be $x_c = 0.575 \dots 0.625$

These must have uniform front tube spacing. I think 32 tubes would be more than enough.

- Mark

Return-Path: drela
 Received: from localhost by henry; Sun, 10 Dec 1995 13:16:47 -0500
 Id AA-9303
 Message-ID: <9512101316.M003@henry>

To: Al Bowers <bowers@wilbur.dfrn.nasa.gov>
 Cc: drela@mit.edu
 Subject: Re: LFRG m137

In-Reply-To: Your message of "Fri, 08 Dec 25 13:26:23 EST."
 Date: Sun, 10 Dec 25 20:16:47 -0500
 From: drela
 X-Mts: smtp

Al,

Return-Path: drela
 Received: from localhost by henry; Sun, 10 Dec 13:36:25 1995
 Id AA-9374
 Message-ID: <9512101336.A003@henry>

To: Al Bowers <bowers@wilbur.dfrn.nasa.gov>
 Cc: drela@mit.edu
 Subject: Re: LFRG m137

In-Reply-To: Your message of "Tue, 26 Nov 95 14:13:44 EST."
 Date: Mon, 25 Dec 95 20:03:36 -0500
 From: drela
 X-Mts: smtp

Al,

I've got a reasonable estimate for the rake location and size:

$x_c = 1.35$
 $\Delta x_c = 0.05 \dots 0.10$

dense ports
 sparse ports

The actual wake thickness at that x_c location might be anywhere from $\Delta x_c = 0.05$ to 0.10 , but it moves up and down with alpha. The "dense" x_c range above will catch it over all reasonable operating points, and that's where we want a relatively dense set of rake ports, that's what I mean by dense. The sparse port set above and below the dense range is mainly to make sure that all the wake is covered. In the higher Mach operating points, there is also a weak shock wave which goes up to about $x_c = 0.30$, but this has a very small contribution to the total momentum defect. The delta x_c in this shock wake is less than 0.01 , which is probably too small to measure reliably, although I would make the integrating rakes cover the range

still, the $x_c = 1.3$ location is fairly conservative, you can move it forward to $x_c = 1.2$ if it makes things easier. The rake x_c dimensions and location should be about the same at that location.

Yesterday, we had all sorts of rain coming down all day, so I wrote up

Mark's Compleat Guide to Integrating Rakes

Somewhat belatedly, I got pages after I got really going. The congressibility-correction stuff got really ugly. Anyways, I'll send it separately, in Postscript form. I would only seriously look at Sections 1 and 3.

Sections 2 and 4 are only important if you need absolute rather than relative measurements, which is not the case here. The bottom line that we're interested in is equation (12). Equations (4) and (5) are the relevant sizing criteria you should consider.

- Mark
 - wake to relax reasonably well. If you normally place rakes at $x/c = 1.55$, then this airfoil would sail for $x/c = 1.15$ or more.

Putting the integrating rakes at the trailing edge would be OK in theory, but gain you have the problem of negligibly-small velocities giving poor accuracy. The integrating rake does "see" low-velocity regions quite well, but this contributes into delta^{*} much more than theta. So the sum theta + delta is accurate, but you can't be sure that theta itself is captured well enough, since this may be masked by delta. And theta is what counts, ultimately. Also, it would be nice to have the integrating rates at the same x/c as the drag rake, so they can be compared. Is it a problem to mount the integrating rakes on slugs cantilevered from the trailing edge? It seems this shouldn't be that hard to do.

- Mark

Al,

Return-Path: drela
 Received from localhost by henry - 5.65.1.1.5.5 04 Apr 25 2001
 id AA1253; Mon, 11 Dec 1995 17:14:32 -0500
 Message-ID: <9512111901AA1253@henry.drela.mit.edu>
 To: Al Bowers <bowers@wincat.drela.mit.edu>
 cc: drela@mit.edu
 Subject: Re: airfoil
 In-Reply-To: Your message of "Mon, 11 Dec 25 11:51:31 EST."
 <9512111901AA1253@libur.drela.mit.edu>
 Date: Mon, 11 Dec 25 10:11:32 -0500
 From: drela
 X-Nts: smtp

Jeff,

I laid out a reasonable rake geometry for 26 tubes, for $x/c = 1.2$ or 1.3. I'll send a full-size Postscript drawing of it separately. My brain-damaged email handler can't insert files. You'll notice that I divided up the x-interval into 26t intervals of varying widths, and then placed a tube at the center of each interval. This allows the simplest-possible midpoint-rule integration with each 26t simply weighted by its interval width:

$$\text{Integral} = \sum_{i=1}^{26} \rho_{i-1} \delta x_i \frac{\partial P}{\partial x}(x_i)$$

Besides the normal-pressure gradients I discussed previously, another argument against placing the rake too close to the trailing edge is that the wake centerline velocity is quite close to zero there, at least for this airfoil. This means that the total pressure signature there will be very small, and you won't be able to measure the local momentum defect very accurately. For example, at one particular operating point of interest (Re=230k, $M=0.65$, $\alpha=1.1^\circ$), the wake centerline velocity at $x/c = 1.1$ is $u/c = 0.1$. The rake tube at that location will see a total pressure of

$$P_0 = P_\infty + 0.61 / 0.5 \rho c U^2$$

which is darn close to $P_0 = P_\infty$. In other words, it's very hard to measure the difference between $u/c = 0.1$ and $u/c = 0.0$, but this difference is quite significant to the integrated momentum thickness and drag. You want to measure the wake after its minimum velocities have reached reasonable levels -- at least $u/c = 0.2$ or more.

Remember that the wake thickness on this airfoil is several times larger than on typical high-Re airfoils. This means that you have to go back proportionately farther back to airoff the

- wake to relax reasonably well. If you normally place rakes at $x/c = 1.55$, then this airfoil would sail for $x/c = 1.15$ or more.

Putting the integrating rakes at the trailing edge would be OK in theory, but gain you have the problem of negligibly-small velocities giving poor accuracy. The integrating rake does "see" low-velocity regions quite well, but this contributes into delta^{*} much more than theta. So the sum theta + delta is accurate, but you can't be sure that theta itself is captured well enough, since this may be masked by delta. And theta is what counts, ultimately. Also, it would be nice to have the integrating rates at the same x/c as the drag rake, so they can be compared. Is it a problem to mount the integrating rakes on slugs cantilevered from the trailing edge? It seems this shouldn't be that hard to do.

- Mark

Return-Path: drela@henry.MIT.EDU
 Received: from PACIFIC-CARRIER.ANDREW.MIT.EDU by henry - 5.65.1.1.5.5 04 Apr 25 2001
 id AA1253; Fri, 13 Jan 1995 22:57:15 -0500
 Received: from HENRY.MIT.EDU by MIT.EDU with SMTP
 id AA1253; Fri, 13 Jan 1995 22:57:27 EST
 Received: from Localhost by henry - 5.65.1.1.5.5 04 Apr 25 2001
 id AA1253; Fri, 13 Jan 1995 22:58:47 -0500
 Message-ID: <9501200316AA1253@henry>
 To: Jeff_Bauer@mgate.drc.nasa.gov;
 cc: bowers@libur.drc.nasa.gov; drela@MIT.EDU
 Subject: Integrating rake
 Date: Fri, 13 Jan 95 22:58:48 -0500
 From: drela@henry.MIT.EDU
 X-Mts: smpt

The January 3 APEX Engineering Summary: see:
 *Jeff Bauer: Mark Drela will be defining the rake..."
 I just want to let you know that I've already sent a full-size layout of the wake rake to Al Bowers.
 I also sent Al a writeup on sizing of the integrating rake tubes, etc.

Since I didn't see Al listed in the summary, I assume that he wasn't at the meeting, so I'm letting you know directly. If what I sent isn't enough, let me know.
 - Mark

Return-Path: drela
 Received: from localhost by henry - 5.65.1.1.5.5 04 Apr 25 2001

From: Sun, 4 Feb 1996 11:53:54 -0500
 Message-ID: <32C4175CA05710@henry>
 To: Al Bowers <bowers@mit.edu>
 Subject: Re: AEW FPA
 In-Reply-To: Your message of "Thu, 21 Feb 1996 10:06 EST."
 Date: Sun, 24 Feb 26 15:53:53 -0500
 From: drela
 X-Mailer: smtp

Al,

Re: Your questions...

>i they have located a 3FS antenna near the spar in the upper surface
 >at about mid-span on both wings'.
 >UGH!!

This is TERRIBLY serious. I would be very weary of putting
 anything on the front upper surface of the airfoil.
 At low Re, near 1max, it takes a very small perturbation
 to really kick up the flow. I have firsthand experience
 in this...

On our Monarch FPA, we had a strut protruding from the
 top of the 3-foot chord wing at 25%. The strut was
 airfoil shaped, and was a mere 6% chord and 1% thick.
 Tufts showed that it caused a separation wedge which
 was 2 feet wide at the trailing edge! I expect AEP
 to be even more prone to this because the transonic flow
 on top makes all blockage disturbances propagate
 sideways along the span with little attenuation.

If the antennas stay where they are, they will seriously
 taint any data you get from the instrument station.

Moving them to the tips would be a good solution.

>z, the builder is making up the wing skins with a lower surface that
 >wraps forward over the leading edge, and then has a slip joint to the
 >upper surface.

I would prefer the seam to be on the lower surface, but
 I don't think that this is that big a deal. At significant
 Mach numbers, the upper flow forward of 5% is still strongly
 favorable, even at high CL. If it will hold things up
 or cost significantly more money, I wouldn't move the seam.

>> we talked sometime ago about detwisting the wing fences as a way to
 >hang the rakes and their associated support structure. I wanted to
 >clarify that, indeed, we do not want any fences, how about lower
 >surface fences only? Most of our experiment is on the upper surface,
 >and that would still give adequate structure to hang the hardware off of.
 Yes, I now think that fences are too risky. Since there
 will be some spanwise variation due to the untwisted

rectangular wing, I expect that there will be some spanwise
 flow near the trailing edge and maybe in the bubbles.
 A constant spanwise flow in itself has little effect on
 the lift characteristics, infinite sweep theory is what
 matters is the spanwise gradient of the spanwise flow,
 which a fence would aggravate. In other words, if there
 is any spanwise flow, you want it to just let it come
 uniformly across the test section without trying to
 block it with fences. If you do block it, it will just
 pile up against the fence and artificially thicken the
 BL there. At the "upstream" fence the BL will thin out,
 which is equally bad.

I would go with the bottom-mounted fences. These don't
 have to be thin, fairly sheet metal, by the way. A thin
 version of a jetliner flag-track fairing would be OK.
 Even an open tube would work fine, and would certainly
 be lighter than anything else. You could mount a
 permanent half-circle cradle on the wing bottom.
 The tube could just bolt down on this, and thus give
 a simple means to recenter the rake's fore-and-aft
 position if this proves necessary.

>From all the computational and experimental evidence
 i've seen, putting such junk on the aft bottom of an
 airfoil has little measurable effect anywhere else.

Mark

Return-Path: drela@henry.MIT.EDU
 Received: from PACIFIC-CAPIBER-HENRY.MIT.EDU by henry; 05 Feb 1.1.3.2 06Apr15-020
 Id AA11826; Tue, 5 Feb 1996 16:15:43 -0500
 Received: from HEUNY.MIT.EDU by MIT.EDU with SMTP
 Id AA11833; Tue, 5 Feb 16:15:43 EST
 Received: from localhost by henry; 05 Feb 1.1.3.2 06Apr15-020
 Id AA11835; Tue, 5 Feb 1996 16:15:43 -0500
 Message-ID: <36CC0621IC.WAC25@henry>
 To: Al Bowers <bowers@mit.dire.nas...>;
 cc: drela@MIT.EDU
 Subject: Stanton tubes, mics, etc.
 In-Reply-To: Your message of "Mon, 25 Feb 96 10:28:51 EST."
 <3622051128.A175@avilbur.dire.nasa.gov>
 Date: Tue, 5 Feb 96 16:15:43 -0500
 From: drela@henry.MIT.EDU
 X-Mailer: smtp

Al,

If I had a choice, I'd be more inclined to go for
 the mics rather than the Stanton tubes. The problem
 with a Stanton tube is that to extract it from it
 requires the assumption that you have a standard
 turbulent BL with a normal-looking log layer.
 At our Reynolds numbers, however, the log layer
 is not very distinct, especially since the BL

is stressed so hard. Also, knowing the of precisely wouldn't help all that much in figuring out the behavior of the BL in the pressure-recovery region. The BL dynamics there are dominated by pressure gradients and shear stresses in the outer part of the BL. The wall shear just doesn't have much to say. I'd say having 5 stanton tubes would be more than adequate to give a broad-brush indication of what's happening near the wall. As a first cut, I would put them at 45, 52, 58, 70, 85, 928.

The good thing about the mics is that they specifically give the data which is of interest: TS wave development in the laminar BL and the front part of the bubble, and also the location of transition. It is these things which have the greatest uncertainty in the codes. I would put the mics in the following * chord locations:

25 32 35 42 45 50 52 54 56 58 60 62 64 66 68 70 75 80

We can fine-tune this if you have more-less ports available.

The dense mic region should pinpoint transition fairly well. This should be visible on the CP x1 curve as well, speaking of which, I hope you'll have a reasonable CP port density over 50-70%. That's where most of the action will take place.

Finally, all the Stanton tubes, mics, and CP ports should not be placed upstream of the rake(s). Do you have enough room for everything?

- Mark

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=====
Return-Path: drela
Received: from localhost by henry; 5-65-1-1.8.2.56Apr25-2503PM
id AAC6415; Fri, 9 Feb 1996 00:17:55 -0500
Message-ID: <960209001755.AAC415henry>
To: Al Bowers <albowers@ilcure.difrc.nasa.gov>
Cc: drela@henry.MIT.EDU, drela
Subject: Re: one last time...
In-Reply-To: Your message of "Thu, 08 Feb 96 13:11:14 EST."
<960208131114.14154@ilcure.difrc.nasa.gov>
Date: Fri, 9 Feb 96 00:17:55 -0500
From: drela
X-Mts: smtp
```

Al,

The instrumentation specs look mostly OK. Here are some additions and possible miss:

The drag rake will need two static ports, roughly at the top & bottom of the "fine" region. These should also be offset sideways by 1° or so to avoid interference from the total tubes. The streamwise location of the actual port holes on the static probes should be at the same x/c as the tips of the total tubes. The average of these two static ports will give the local velocity at the wake rake, and the difference will give the transverse pressure gradient and hence the flow curvature.

It seems that the 35 deg wedge exclusion zones may cause a shortage of real estate to place everything. If that's a problem, you may want to move the rakes toward to $x/c = 1.0$. I wouldn't go any farther forward, though.

I'll send a Postscript sketch of the layout I have in mind. The Stanton tubes are the lowest priority, and they are also the most intrusive, so I would place these farthest out.

Note that I'm not counting the pressure ports ahead of 25% chord in the 35 deg rake exclusion zone. This should be OK, since the BL is stable ahead of that location, and the ports will not precipitate a disturbance wedge. This allows a tighter setup.

You have a CP Port at 100% chord on the upper surface? If it makes things easier, this port can face straight back rather than upward. Keeping the TE base pressure would be useful.

Some observations:

Overall, it seems that hanging all the stuff behind the wing will be difficult. You have to hammer into the instrumentation guys the importance of keeping everything light, otherwise there may be a flutter risk. Of course, you can always dump lead into the leading edge to compensate, but that is a last resort.

It seems the crossbar for mounting the rakes will be difficult to make light, stiff, and with a small frontal area. Ideally, it would be a streamlined carbon fiber tube. We've built many such tubes for our hydrofoil boat. The procedure is to take a thin-walled aluminum tube, 0.015" wall or less, flatten it into a suitable ellipse, wrap it with prepreg carbon, wrap it with heat shrink tape, and cook it. When done, you pour in epoxi acid to dissolve the aluminum mandrel. This is much easier than it sounds, and the result is an outrageously strong, stiff, and light streamlined crossbar. I think the aeroenvironment people could frank one out in no time. Alternatively, you can just use

a flattened aluminum tube, but this will be about as heavier for the same stiffness. I think you can buy "streamline tubing" from aircraft spruce or whatever, but I don't know if they have the appropriate sizes. It may also be aluminum rather than some good hard alloy like 2024.

One potential problem that a metal crossbar will have is the thermal expansion mismatch with the wing. In contrast, a carbon crossbar would be matched very well.

I would most likely go with aluminum tubes for the mounting strings. They don't have to be all that small -- up to 1.25" diameter is probably OK. This way you can use a thin wall and save weight.

Enough rambling for now.

Tell Tony that I already surfed his web page.

- Mark

```
Return-Path: drela
Received: from localhost by henry; 5 Apr 1.1.8.2 06Apr25-0253PM
ID: AEE7C5; Fri, 9 Feb 1996 12:50:00 -0500
Message-ID: <9602091552.AA25473@henry>
To: Al Bowers <owers@willbur.dirf.nasa.gov>
Cc: drela@mit.edu, jett.bauer@gsfc.gsfc.nasa.gov
Subject: Cf
```

In Reply-To: Your message of Fri, 31 Mar 14:47:12 EST."

Date: Tue, 05 Mar 12:34:05 -0500

From: drela

X-Mta: smtp

1558 = 258 + 758 at Re = 302K,

and 1558 = 128 + 928 at Re = 203K.

A?

I've made two alternative instrumentation layouts which are more compact than Layout I. I think you should remove the requirement of not having anything in front of the integrating rakes. In fact, it would be beneficial to see if there actually is any disturbance from the pressure taps or whatever. An integrating rake is a cheap way to do this.

I like Layout II the most. Note that it has 4 i-rakes instead of 3. If this is a major problem, then Layout III is passable. It would be nice to have that extra i-rake outside of the pressure-tap spanwise station, though.

- Mark

```
Return-Path: drela
Received: from localhost by henry; 5 Apr 1.1.8.2 06Apr25-0253PM
ID: AEE7C5; Fri, 9 Feb 1996 12:50:00 -0500
Message-ID: <9602091552.AA25473@henry>
To: Al Bowers <owers@willbur.dirf.nasa.gov>
```

Subject: Cf

In Reply-To: Your message of Fri, 31 Mar 14:47:12 EST."

Date: Tue, 05 Mar 12:34:05 -0500

From: drela

X-Mta: smtp

>I ran some numbers for max and min diameter of the >Preston tubes. The max diameter is about 0.615c >and the min diameter is about 0.08c.

Well, shewt.

This leads me to reconsider the whole reason for measuring Cf in the first place -- it is one of the terms in the von Karman momentum equation that makes the BL grow:

$$\frac{d \ln(\theta)}{dx} = \frac{\frac{2f}{2} - H + 2 - M}{\frac{2}{2}} = \frac{H - 2 - M}{dx}$$

It turns out that on the back 50% of the upper surface, the Cf term is fairly minor compared to the d ln(theta) dx Pressure gradient term. At x/c = 0.75, the equation balance is roughly:

$$1558 = 258 + 758 \quad \text{at } Re = 302K,$$

$$\text{and } 1558 = 128 + 928 \quad \text{at } Re = 203K.$$

For a "normal" high-Re airfoil in cruise, it might be $1558 = 738 + 308$ instead. One logical conclusion is that for AEEV it is more important to measure $H + 2 - M$ than Cf, rather than Cf.

The other reason for measuring Cf is to diagnose separation. But again, I think H is a much better indicator of separation than Cf.

Let me offer a crazy idea for a fall-back if the optical gauges don't work out.

Instead of the Cf gauges, we use min/max integrating rates,

These effectively measure $H + 1$, which is enough to get the whole term -- note that θ and Δ are known from δp , overall, I could much rather see $H + \theta$ than δp . You can always directly estimate Δ from H , but in the other way around, since the δp relation is quite flat in strong adverse pressure gradients,

To put it another way: A Preston tube is a sort of a crude integrating "rake" over the lower part of the BL and a Stanton tube goes lower still, near separation, the lower parts of the BL are not all that dynamically important, since most of the action happens higher up -- hence the rationale for the I-rakes.

A potential problem is that the I-rake reservoirs will cause a large separation behind them. This is clearly a risk, but I think that this can be minimized by using a minimal rake reservoir tube and flattening it. I would also place them at $x/c = 0.65, 0.70, 0.80, 1.00$.

I-rakes at $x/c = 0.45, 0.55$ would carry the greatest separation risk by far, so I would leave these off.

In any case, the I-rake in the wake will quantify any problem with separation caused by the surface I-rakes.

- Mark

=====

Return-Path: drela
 Received: from localhost by henry; 5 Apr 11:18:2 26 Apr 95 02:23 PM
 Id AA23823; Tue, 5 Mar 1996 17:39:00 -0500
 Message-ID: <9603052239-AA23823@henry>
 To: Al Bowers <owers@willbur.dir.nasa.gov>
 cc: dreahenry@MIT.EDU, Jeff_Bauer@willbur.dir.nasa.gov
 Subject: Re: Of
 In-Reply-To: Your message of *Tue, 5 Mar 26 10:22:43 EST.*
 <9603051922.AAC4953@willbur.dir.nasa.gov>
 Date: Tue, 25 Mar 96 17:39:50 -0500
 From: drela
 X-Mts: smtp

=====
 Return-Path: drela
 Received: from localhost by henry; 5 Apr 11:18:2 26 Apr 95 02:23 PM
 Id AA23823; Wed, 17 Apr 1996 11:44:49 -0400
 Message-ID: <9604171541-AA23823@henry>
 To: drela
 Subject: Re: microphones...
 In-Reply-To: Your message of *Thu, 11 Apr 26 10:35:56 EDT.*
 <960411154046@willbur.dir.nasa.gov>
 Date: Wed, 17 Apr 96 11:44:49 -0400
 From: drela
 X-Mts: smtp

H is the shape parameter $\Delta\theta$: θ , not the BL height.
 The integrating rake measures the sum $\Delta\theta + \theta$, which is equivalent to

$\Delta\theta + \theta = H + 1 - \theta$ which is in a way it measures $H + 1$.

>But even if we have really well calibrated Preston Stanton tubes, I
>>anticipate that near separated regions, we will see very sluggish
>>flows and have similar problems measuring pressures as if we had the
>>drag rake too close to the TE.

% To: Al Bowers <owers@willbur.dir.nasa.gov>
 Hi Al,

use. Different problem. In the wake you only want theta, since that uniquely determines profile drag. The delta part is a "constraint" which is biggest where there is lots of sluggish flow in the BL or wake. The farther back you go, the less sluggish flow there is, and the less delta you have to correct for.

BUT...

On the surface, it is mainly the $\Delta\theta$ -- that you want. The more sluggish the flow the more effective the I-rake measurement will be!

Would you want a vertical tube at the tip to be sure that we really did achieve free-stream flow and the B-L is entirely embedded in the wake profile? My gut feel is that you would indeed like to have those things, but the totals might be a bit problematic, or maybe not...

Having a separate total tube at the tip might be a good idea, but if you went to the trouble of having the extra data port, I would prefer two stacked I-rakes -- one for the lower and one for the upper BL halves.

Having the two readings would let you deduce what the BL is doing with more certainty. You could still measure the sum $\Delta\theta + \theta$ from the average of the two pressures, but from the difference of the pressures you could estimate the difference $\Delta\theta - \theta$, which is also significant.

If you go with just one single I-rake, it will just need to be taller than the largest anticipated BL thickness. I can estimate the required heights for any given condition. One hassle is that the height of the first I-rake at $x/c = 0.6, 0.7, \dots$, will most likely have to be significantly different. This will preclude mass-production, but that may not be a big deal.

Do you guys have a latest estimate of the anticipated Mach 8.5 conditions you will be testing at. I can't estimate this since I don't have the latest estimates of vehicle weight and the planned test altitudes.

- Mark

=====

Return-Path: drela
 Received: from localhost by henry; 5 Apr 11:18:2 26 Apr 95 02:23 PM
 Id AA23823; Wed, 17 Apr 1996 11:44:49 -0400
 Message-ID: <9604171541-AA23823@henry>
 To: drela
 Subject: Re: microphones...
 In-Reply-To: Your message of *Thu, 11 Apr 26 10:35:56 EDT.*
 <960411154046@willbur.dir.nasa.gov>
 Date: Wed, 17 Apr 96 11:44:49 -0400
 From: drela
 X-Mts: smtp

Yes, it would be nice if the microphones gave its frequency content. The frequencies are quite large, however, mainly due to the high TAT. I'll send you a plot showing the TS amplitudes for a range of frequencies. "L" on the plot is the chord, and " w " is the radial frequency.

For the upper surface, the dominant frequencies are in the range 1500 Hz - 5000 Hz. This is a bit brisk for sampling in time! Is there any way to have an on-board spectrum analyzer that could be sampled at a much more leisurely 1 Hz, say? I don't know if such airborne systems exist, so this may be wishful thinking.

Here's an alternative low-brew approach: Record the microphone outputs on on-board recorders, and play it back in the lab. I don't know how good commercial Walkman-level machines are, but 1500-2000 Hz is in the upper range of music, so they should be OK. Considering how cheap such recorders are, it's something to look at. You have two recorders per microphone for better reliability.

- Mark

sets the tolerances for these pressures

- * Normal forces, 5.1 g is too big if you want to control the M=5.0 better than 1% during a 1-g run!
- * Mach 1.0, 5.0 or even 6.0 would be better.
- * Alpha = 2.1 deg seems like a lot for transonic flow. Is 2.1 deg possible?

The L-rakes layout I sent a while back should be more than adequate to capture the whole wake. Even if it misses the edge a bit, so what? We can still compare the L-rake reading with what it should see base on the real-rake measurements. This will still settling 2-Dness for lack thereof.

- Mark

```
Return-Path: drela
Received: from localhost by henry; 15.65.1.1.8.2 06APR95-0203PM
Id: A1CC06; Tue, 23 Apr 1996 23:23:16 -0400
Message-Id: <9604232323.A117915@mail.bur.dfrc.nasa.gov>
To: Al Bowers <bowers@silbur.dfrc.nasa.gov>
Cc: drela@henry.mit.edu
Subject: Re: Sir!
```

In-Reply-To: your message of "Tue, 23 Apr 96 15:39:26 EDT."
Date: Tue, 23 Apr 96 23:23:16 -0400
From: drela
X-Mts: smtp

Hi Al,

I looked over your instrumentation spec sheet.

I asked our local turbulence-measurement guru, Kenny Breuer about the required mic sensitivity, since I wouldn't even hazard a guess. He's in Sweden, but I expect an email answer any minute now...

All the tolerances look adequate except...

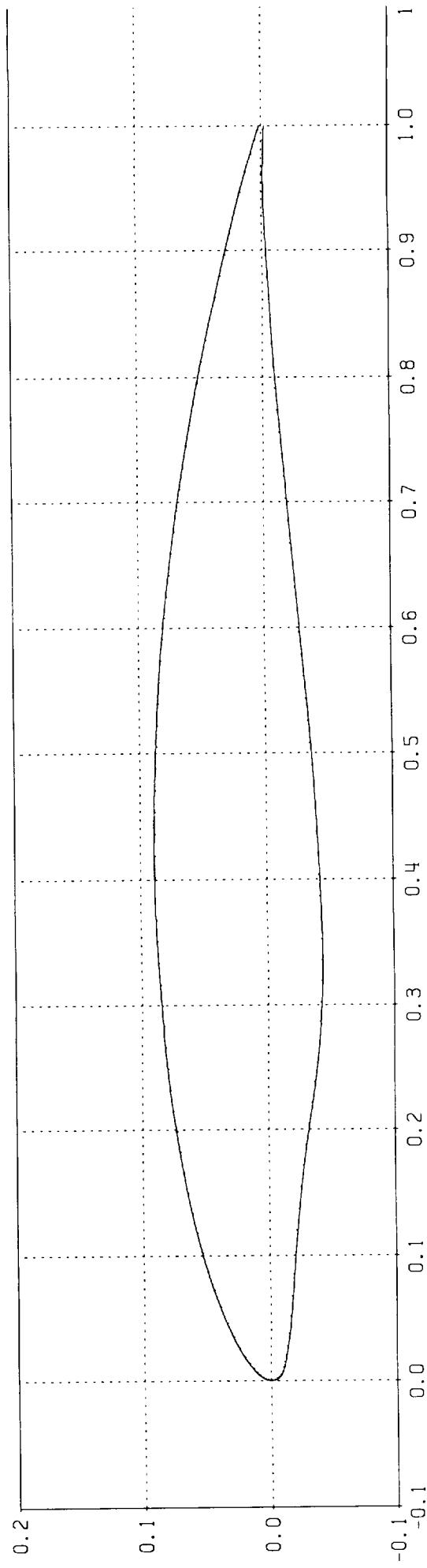
- * Mach #. *+/- 0.5% is a bit too big, 0.2% or less would be better. Since you can get Mach directly from the ratios Ftotal/Fstatic, Fstatic, the uncertainty in Mach is the same as the uncertainty in the pitot pressure Ftotal/Fstatic. This also

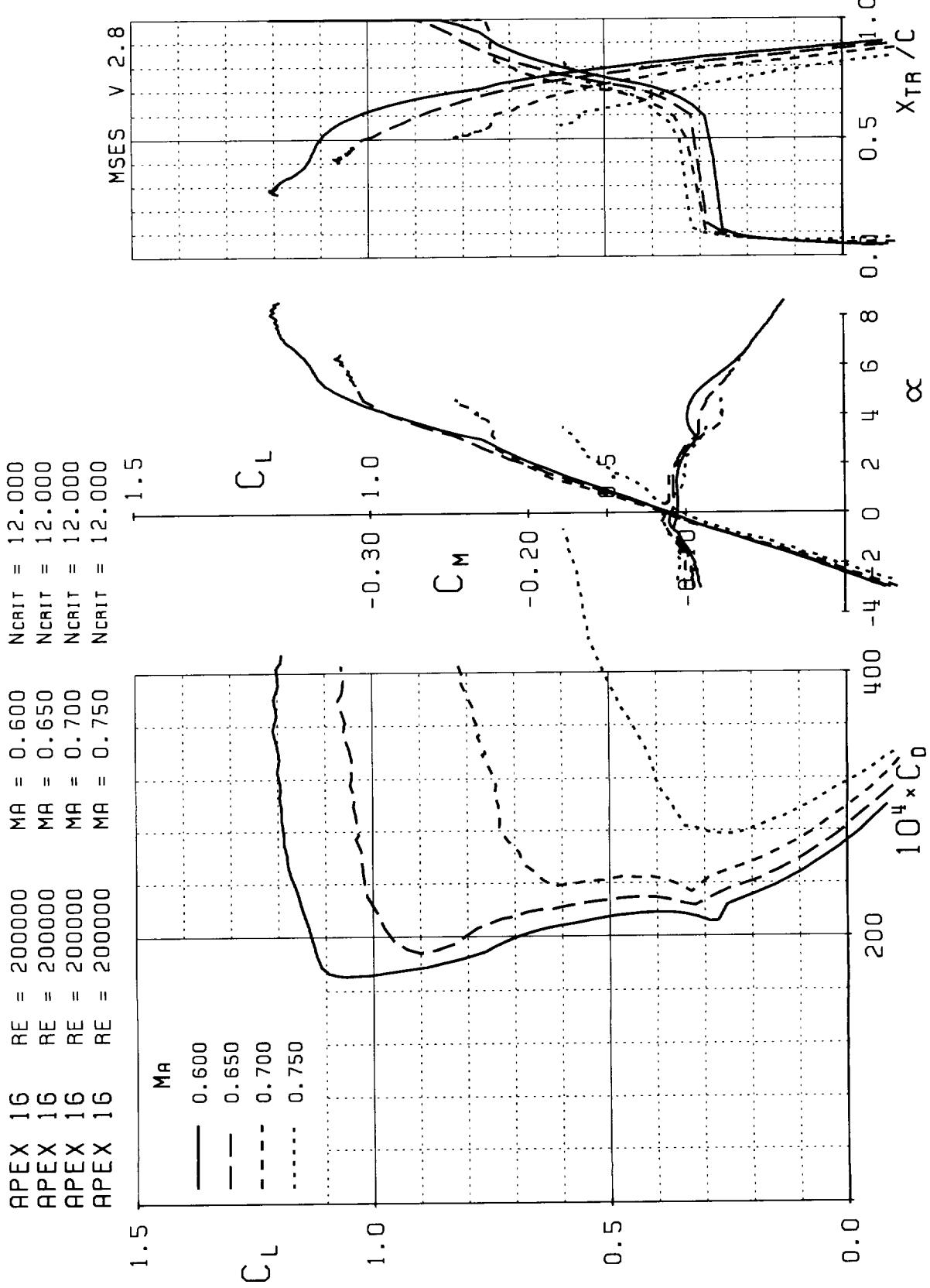
Appendix B: Key figures and sketches

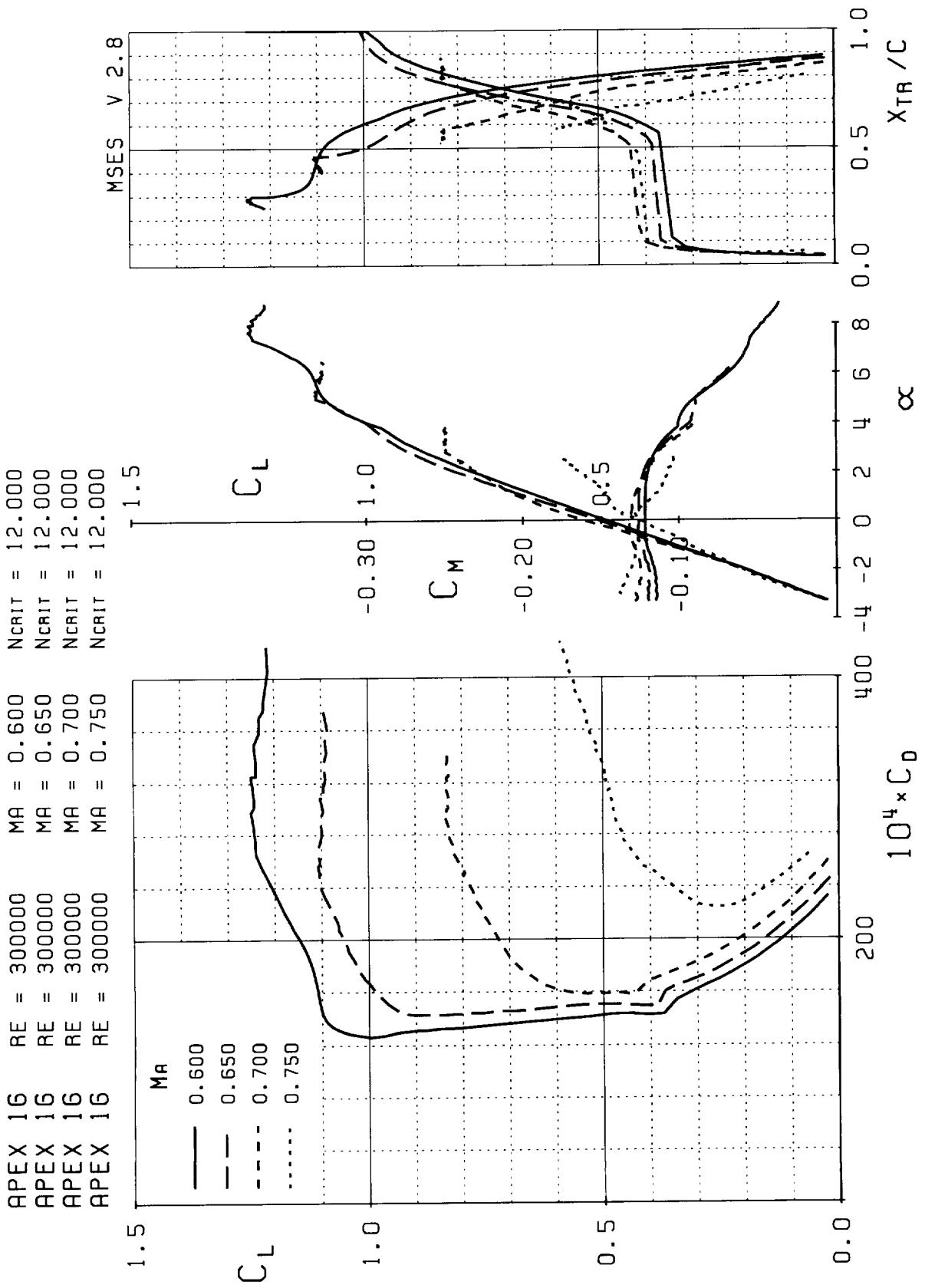
APEX 16

x	y	x	y	x	y
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0.996936	0.002939	0.182124	0.071014	0.335659	-0.042958
0.988795	0.005338	0.165761	0.068148	0.352439	-0.042918
0.977291	0.008683	0.149557	0.065024	0.369357	-0.042645
0.962745	0.012817	0.133556	0.061628	0.386421	-0.042174
0.946684	0.017254	0.117799	0.057944	0.403596	-0.041531
0.930138	0.021685	0.102355	0.053956	0.420860	-0.040737
0.913447	0.026011	0.087310	0.049650	0.438200	-0.039812
0.896699	0.030208	0.072785	0.045017	0.455601	-0.038770
0.879909	0.034271	0.058990	0.040088	0.473059	-0.037626
0.863091	0.038196	0.046262	0.034955	0.490567	-0.036393
0.846244	0.041983	0.035096	0.029839	0.508116	-0.035082
0.829370	0.045629	0.025944	0.025057	0.525706	-0.033703
0.812468	0.049137	0.018879	0.020836	0.543327	-0.032267
0.795546	0.052501	0.013576	0.017218	0.560976	-0.030780
0.778591	0.055724	0.009594	0.014109	0.578654	-0.029249
0.761610	0.058805	0.006567	0.011391	0.596354	-0.027682
0.744595	0.061747	0.004250	0.008957	0.614079	-0.026084
0.727557	0.064550	0.002489	0.006718	0.631831	-0.024461
0.710498	0.067212	0.001204	0.004607	0.649604	-0.022820
0.693423	0.069734	0.000381	0.002575	0.667405	-0.021167
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0.590773	0.081760	0.004935	-0.008327	0.773800	-0.011521
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0.453510	0.089073	0.068128	-0.017542	0.913086	-0.002205
0.436337	0.089267	0.085370	-0.018957	0.930043	-0.001634
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0.402019	0.089166	0.120161	-0.021923	0.962660	-0.001176
0.384874	0.088867	0.137275	-0.023559	0.977023	-0.001319
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0.299492	0.084791	0.221631	-0.034644		
0.282523	0.083434	0.238067	-0.036998		
0.265599	0.081885	0.254021	-0.038916		
0.248739	0.080140	0.269935	-0.040388		
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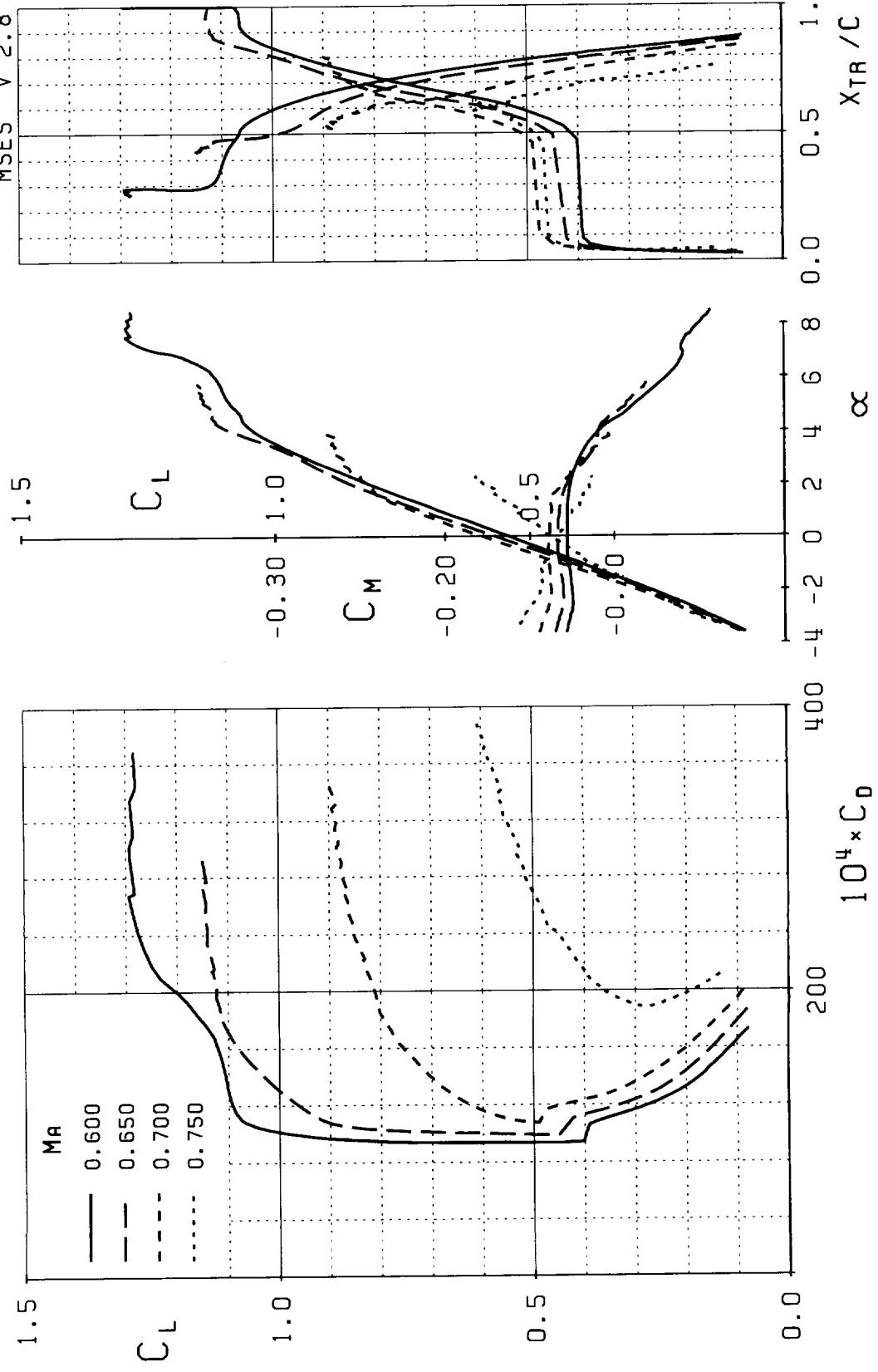
APEX-16 Airfoil
13.11 % max thickness
2.68 % max camber

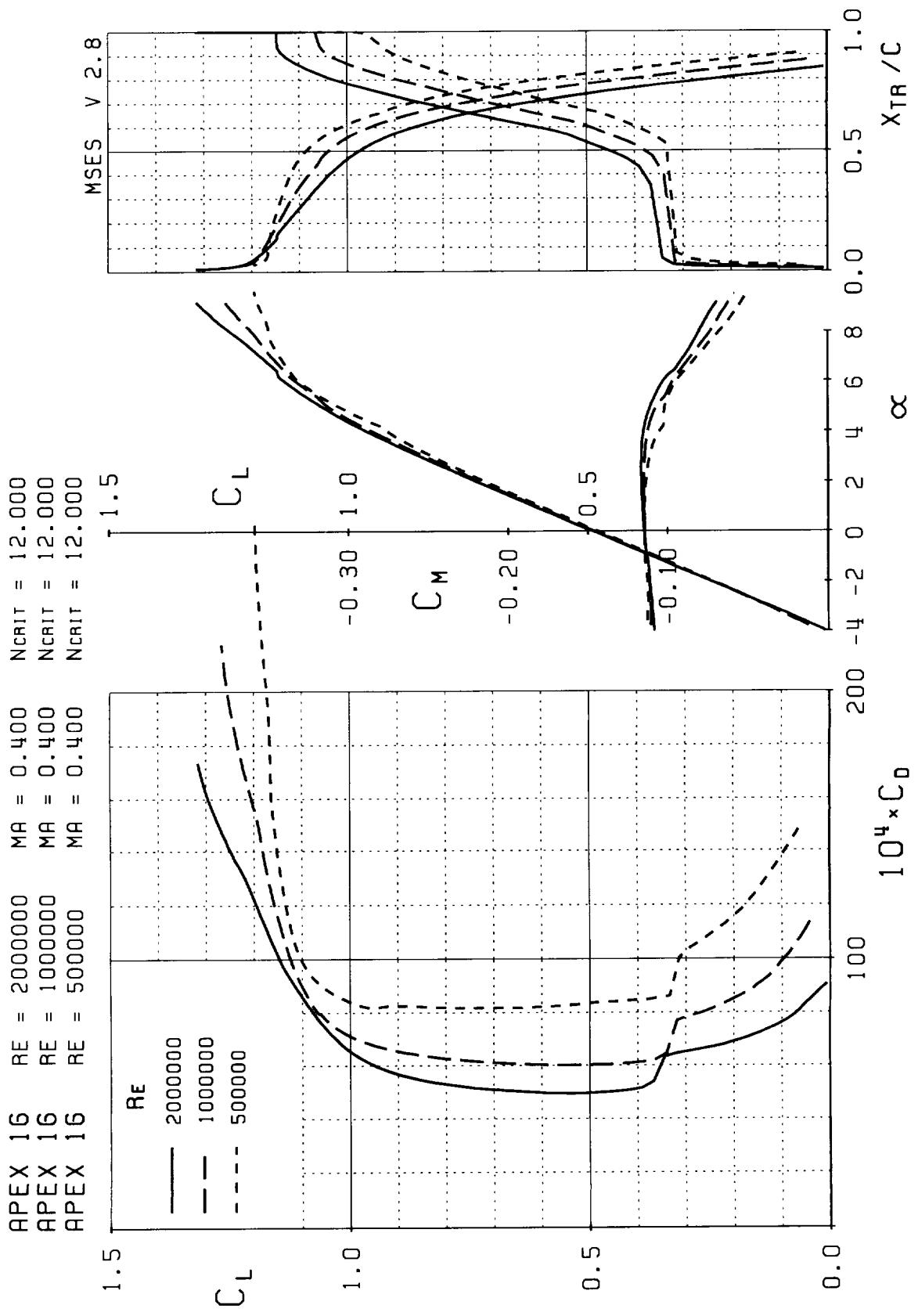




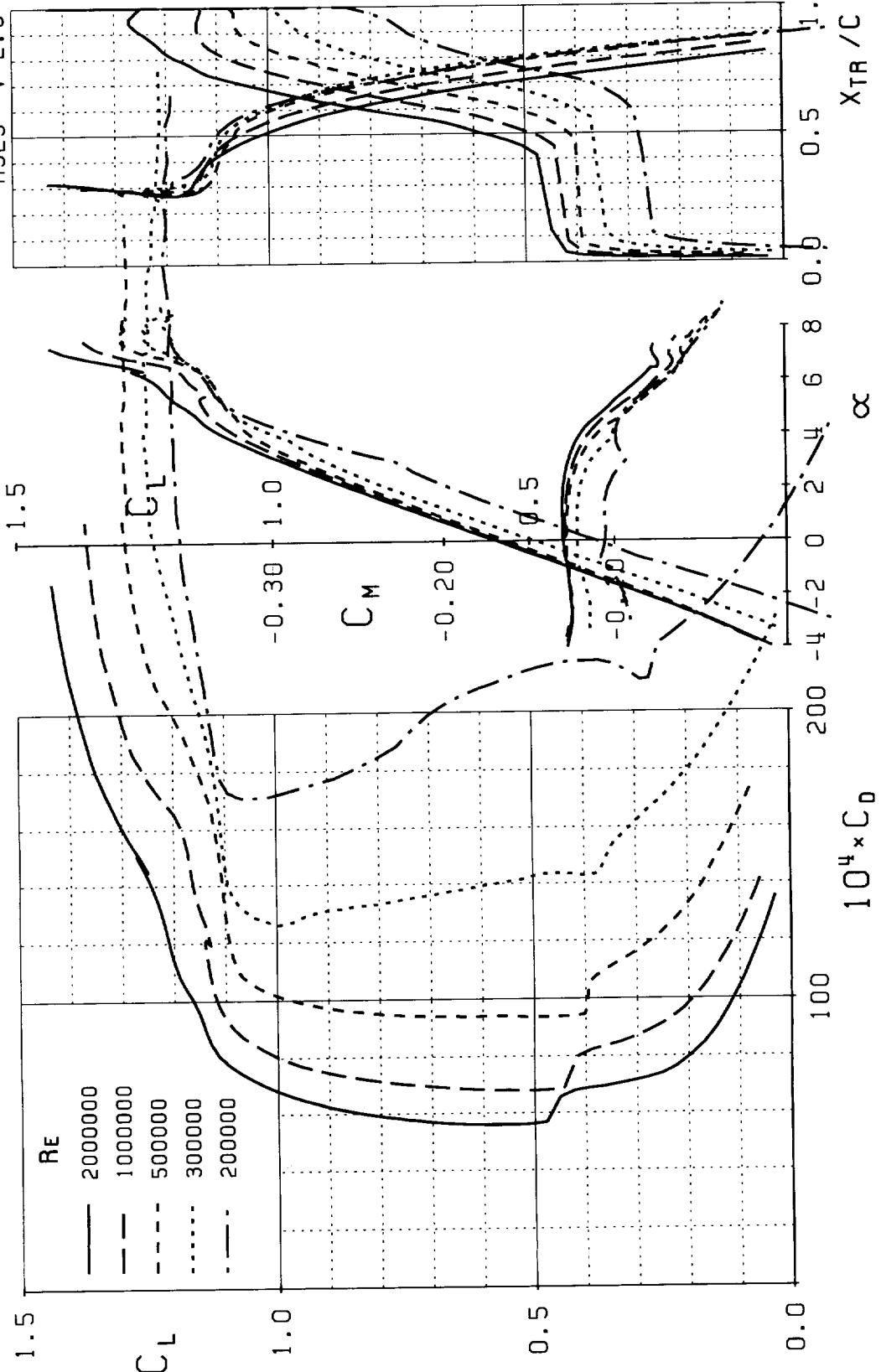


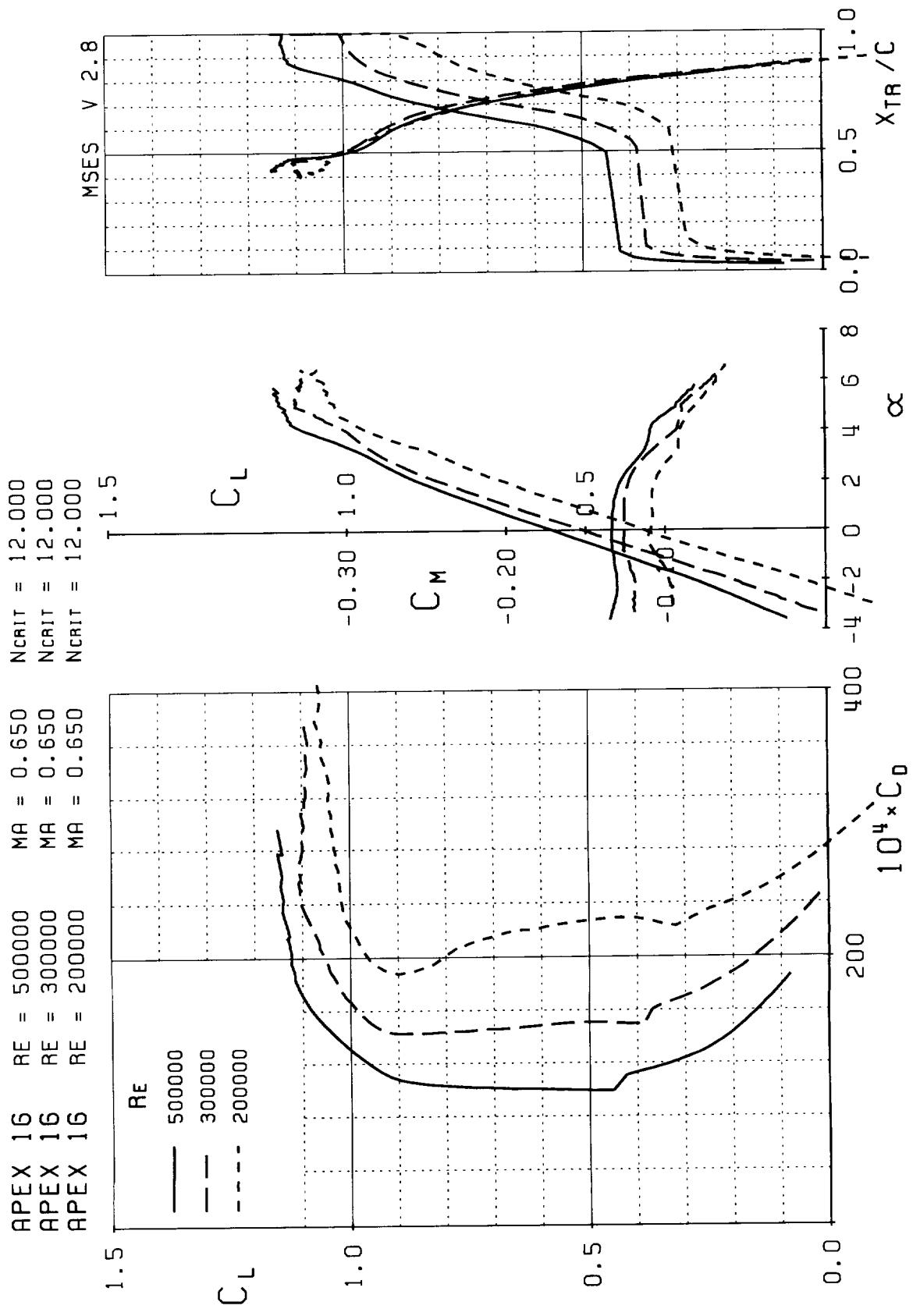
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 APEX 16 RE = 500000 MA = 0.650 NCrit = 12.000
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 APEX 16 RE = 500000 MA = 0.750 NCrit = 12.000

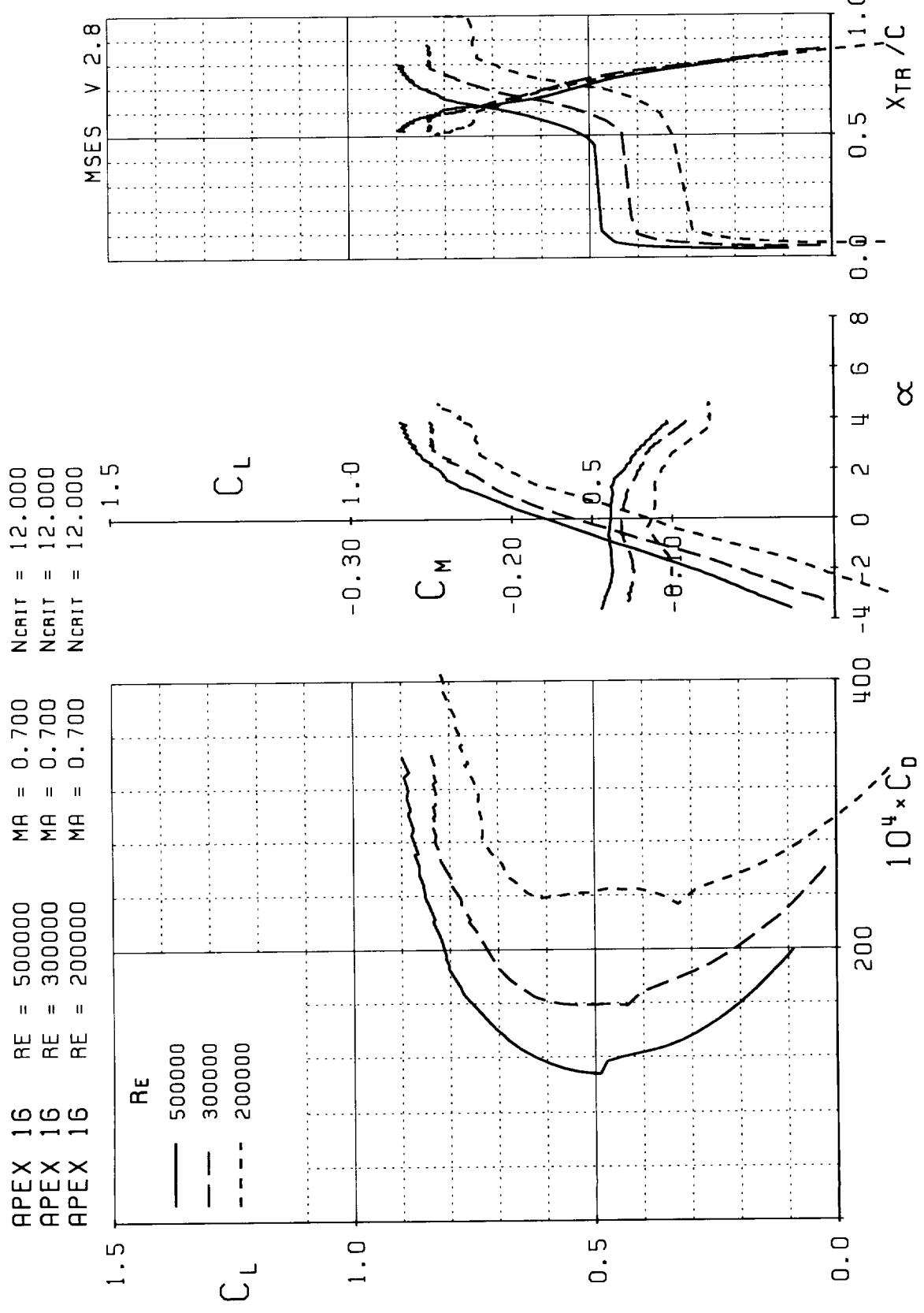


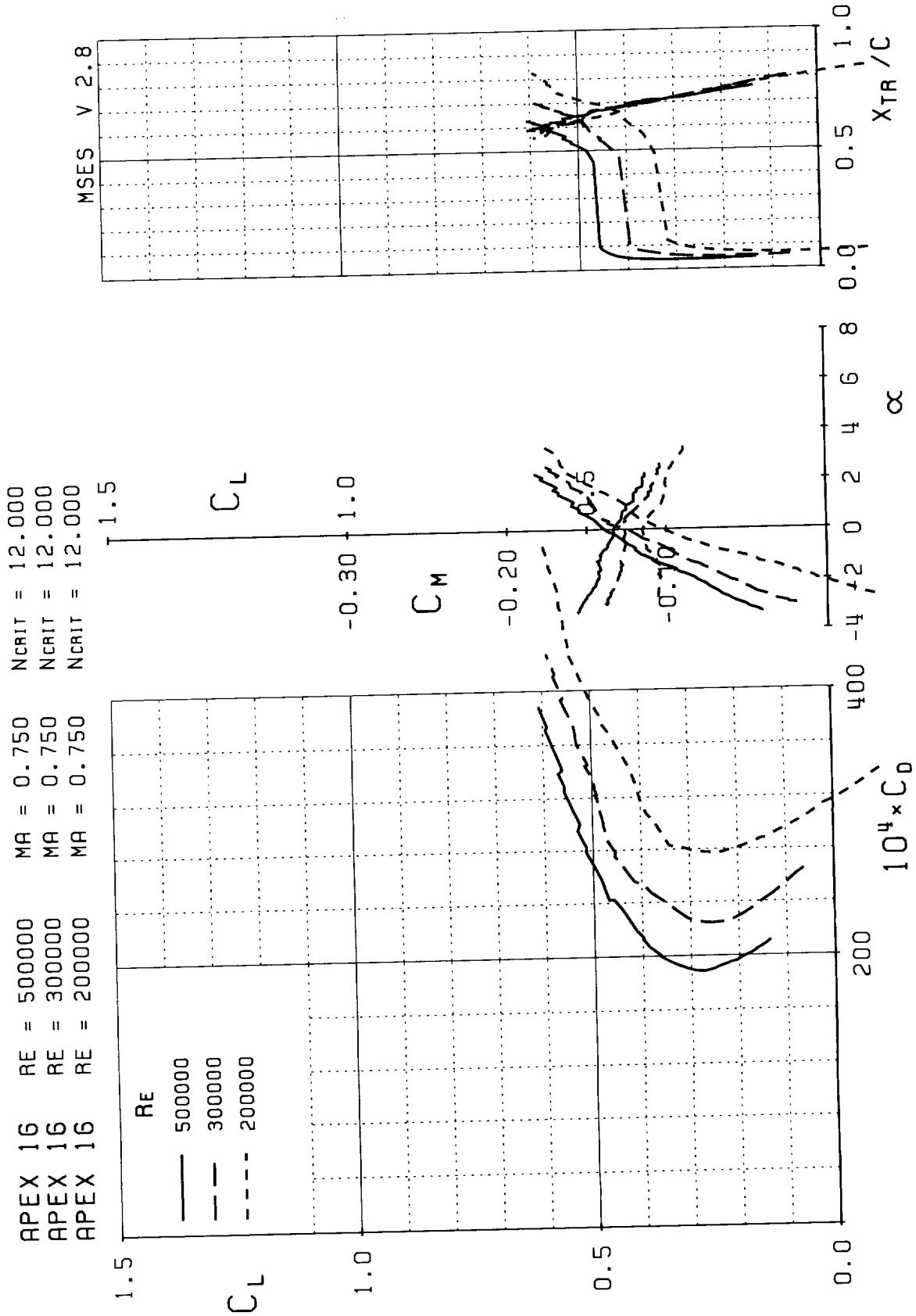


APEX 16 RE = 2000000 MA = 0.600 NCRT = 12.000
 APEX 16 RE = 1000000 MA = 0.600 NCRT = 12.000
 APEX 16 RE = 500000 MA = 0.600 NCRT = 12.000
 APEX 16 RE = 300000 MA = 0.600 NCRT = 12.000
 APEX 16 RE = 200000 MA = 0.600 NCRT = 12.000









$RE = 200$

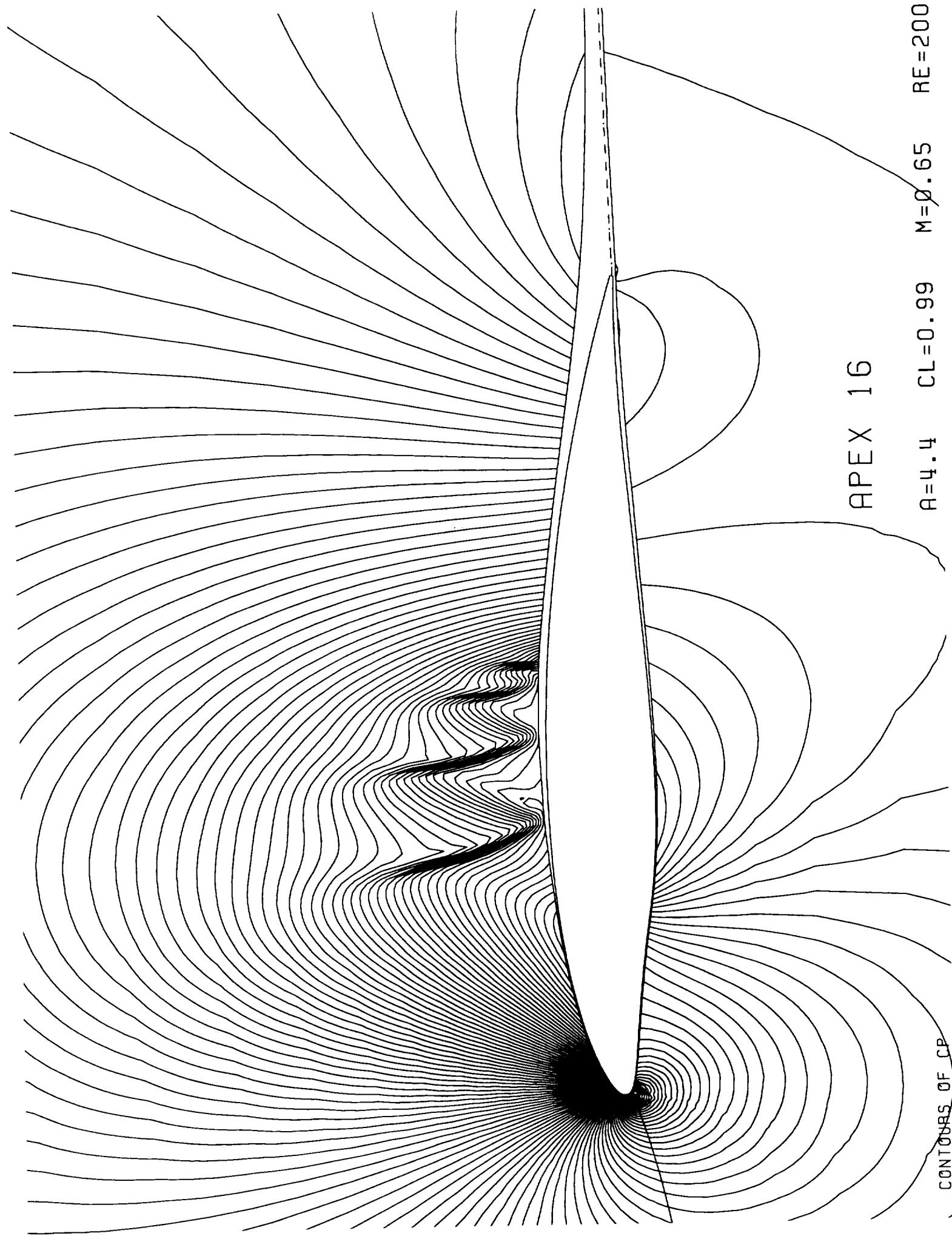
$M = \beta = 0.65$

$CL = 0.99$

$A = 4.4$

APEX 16

CONTours OF CP



BL VELOCITY PROFILES

$R = 4.4$ $C_L = 0.99$ $M = 0.65$ $RE = 200K$

APEX 16

